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Dynamic User Equilibrium Model for Combined Activity-Travel Choices Using Activity-Travel Supernetwork Representation

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16. Abstract

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Abstract Integrated urban transportation models have several benefits over sequential models including consistent solutions, quicker convergence, and more realistic representation of behavior. Static models have been integrated using the concept of Supernetworks. However integrated dynamic transport models are less common. In this paper, activity location, time of participation, duration, and route choice decisions are jointly modeled in a single unified dynamic framework referred to as Activity-Travel Networks (ATNs). ATNs is a type of Supernetwork where virtual links representing activity choices are added to augment the travel network to represent additional choice dimensions. Each route in the augmented network represents a set of travel and activity arcs. Therefore, choosing a route is analogous to choosing an activity location, duration, time of participation, and travel route. A cell-based transmission model (CTM) is embedded to capture the traffic flow dynamics. The dynamic user equilibrium (DUE) behavior requires that all used routes (activity-travel sequences) provide equal and greater utility compared to unused routes. An equivalent variational inequality problem is obtained. A solution method based on route-swapping algorithm is tested on a hypothetical network under different demand levels and parameter assumptions.

Keywords Integrated urban transport model · Activity-Travel Networks · Dynamic user equilibrium · Route-Swapping Algorithm

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1 Introduction

Urban transport modeling involves several dimensions of individual choice including activity participation, location, time of participation, duration, choice of mode, and route. Often the choice models are sequentially applied with feedback: initially, the choice environment is assumed fixed and the individual choices are determined. Subsequently, given the individual choices the choice environment is adjusted. If feedback is involved, the two steps are repeated until the individual choices and the resulting choice environment are in equilibrium. We also refer to this state as converged solution. This process of iteratively solving a sequence of models forms the basis of the four-step urban transportation modeling paradigm.

As opposed to the sequential procedure, several studies have explored integrated choice models particularly with respect to static transport models. Integrated urban transportation models have several benefits over sequential models including consistent solutions, quicker convergence, and more realistic representation of behavior. Static urban transport models have been integrated using the concept of Supernetworks ((Sheffi, 1985); also referred to as Hypernetworks, (Sheffi and Daganzo, 1979, 1980)). However integrated dynamic transport models are less common.

Dynamic traffic assignment models (Peeta and Ziliaskopoulos, 2001) have been developed over the past two decades and have addressed several of the short-comings of the static traffic assignment procedures. In particular, DTA models have increased traffic flow and behavior realism and the explicit modeling of time-varying flows. These advantages allow DTA to be applied to real-time traffic management, ATIS, and other ITS measures (Mahmassani, 2001; Ben-Akiva et al, 2001). While traditionally DTA models were restricted to determining route choices given an exogenous time-sliced demand matrix, more recently DTA models that capture two choice dimensions - route and departure time choice - have been developed (Friesz et al, 1993; Ran et al, 1996; Huang and Lam, 2002; Wie et al, 2002; Szeto and Lo, 2004; Zhang and Zhang, 2007). To capture behavioral realism better there is a need to consider additional choice dimensions within a dynamic traffic assignment framework.

Initial work toward integrating additional choice dimensions in DTA models are (Abdelghany et al, 2001, 2003). Abdelghany et al (2001) develop dynamic spatial microassignment procedures when the unit of analysis is trip chains instead of trips. However, they do not model additional choice dimensions such as departure time, activity location and duration endogenously. Abdelghany et al (2003) addresses a more general choice problem. They determine the departure time, route choice and the sequence of activities simultaneously.

More recent studies in integrated dynamic models include Lam and Huang (2003); Zhang et al (2005); Kim et al (2006); Rieser et al (2007). Lam and Huang (2003) develop a dynamic equilibrium model considering activity location, route, and departure time dimensions. Their framework, however, assumes the duration of activity participation as exogenous. Capturing activity duration is essential to understand the effect of activity scheduling on traffic congestion. An integrated work activity scheduling and departure time choice model in a network with bottleneck congestion is developed by Zhang et al (2005). However, they consider single activity participation only. A logical extension is to consider multiple activities and activity chaining decisions. This is the focus of the paper by Kim et al (2006). They present an activity chaining model formulated from the perspective of a time use problem with budget constraints. Their model includes a dynamic traffic assignment simulation model to obtain network travel times and an iterative day-to-day dynamic process where activity chains are updated based on the network travel times computed in previous iteration. Whether such an iterative procedure results in consistent solutions and the performance of the solutions compared to more holistic frameworks are interesting research questions that merit attention. Rieser et al (2007) describe a multi-agent simulation (MATSim) that takes individuals complete activity sequence as input. Individual's behavior in terms of their route choice and departure time choice are determined iteratively with a traffic flow simulator. They describe a conceptual framework to extend the MATSim to incorporate activity rescheduling and participation decisions.

In this paper, activity location, time of participation, duration, and route choice decisions are jointly modeled in a single unified dynamic framework referred to as Activity-Travel Networks (ATNs). The proposed integrated framework is motivated by the following considerations: (a) to capture activity demand-supply dynamics in addition to transportation demand-supply dynamics, and (b) to obtain a consistent equilibrium solution across all dimensions of choice. ATNs is a type of Supernetwork where virtual links representing activity choices are added to augment the travel network to represent additional choice dimensions. Each route in the augmented network represents a set of travel and activity arcs. Therefore, choosing a route is analogous to choosing an activity location, duration, time of participation and travel route.

2 ATN Representation and Motivation

ATNs use a network representation where nodes are activity centers that are joined by travel links. Activities are represented by arcs that both originate and terminate in the same node (activity centers). Each activity arc is characterized by a unique activity type and a set of durations. An activity-travel sequence for an individual can be represented as a 'route' that includes both travel and activity arcs. All individuals at the beginning of the model start from 'home' and must participate in a predefined set of activities. All activity-travel sequences that traverses the set of activity arcs in which an individual participates in are considered feasible sequences. The model time frame may be set arbitrarily and is presented in a discrete-time setting. Durations of arc-traversal for travel arcs is always assumed to be a function of flow, while for activity arcs it is assumed fixed. Consistent with rational behavior assumption, each individual chooses the activity-travel sequence that provides the maximum generalized utility. However, modeling the network dynamics at an individual level is computationally intensive. Therefore, we treat all individuals residing in the same 'home' node, who participate in the same set of activities as similar. We accordingly modify the behavioral framework to be consistent with Wardrop's (Wardrop, 1952) equilibrium framework. The behavioral rule adopted is 'all used routes (activity-travel sequences) provide equal and greater utility compared to unused routes'. In other words, at equilibrium no individual can improve her utility by unilaterally changing her travel choice decisions.

A primary motivation of the ATNs representation is to capture the effect of activity and transportation demand-supply dynamics in travel choice decisions. Consider a hypothetical scenario in the double-diamond network shown in figure 1. The network consists of eight nodes: Home node (H), Work node (W), four Non-work activity centers (N1-N4), and two intermediate nodes (I1 and I2). The nodes are connected by twelve arcs: 3, 4, 10, and 11 are the activity arcs and the rest are travel arcs. Let us call the diamond with the home node as the residential neighborhood diamond (R-diamond) and the other as business neighborhood diamond (B-diamond). The total demand for travel from home to work is 100 individuals; all individuals drive alone to work. Further, 50 individuals drive directly from home to work while 50 individuals make a stop to participate in a non-work activity en route to work. All individuals have to arrive at work at the same time, (say) T. All travel arcs have a capacity of 50 vehicles per time unit and free-flow traversal time of one time unit, while the duration of non-work activity participation (which is also the time for traversal of activity arc) is two time units. The utility of participating in the non-work activity is 100 utils (let utils be the unit of measuring utility) while the utility of travel on an arc is -5*(travel time) utils. As mentioned earlier, the travel arcs have fixed capacities: at free-flow a travel arc traversal would fetch -5 utils, while a queuing delay by one time unit would result in a payoff of -10 utils.

There exist two possible activity-chain sequence in the double-diamond network: i) Home to Work, and ii) Home to Non-work activity to Work. The former can be accessed via four different paths while the latter has eight paths - four paths each that visit a non-work activity center in R-diamond and B-diamond. Since the utility of participating in the non-work activity in all four nodes is the same, based on traditional models of utility maximization, they attract equal amount of traffic. Therefore such an assignment model would result in each of the eight paths that pass through the non-work activity having a flow of 50/8. The corresponding total free-flow traversal time is 7 time units (therefore start time is $(T-7)^{th}$ time unit). For the individuals who drive straight to work, the traversal time is 5 time units; the corresponding flow is divided among the four paths (50/4). However, link 7, with a capacity of 50 vehicles per time unit, has an upstream demand of 75 vehicles at the start of $(T-3)^{th}$ time unit. This leads to delay by one time unit for 25 individuals and a loss in overall utility of 125 utils (assuming there is no late arrival penalty).

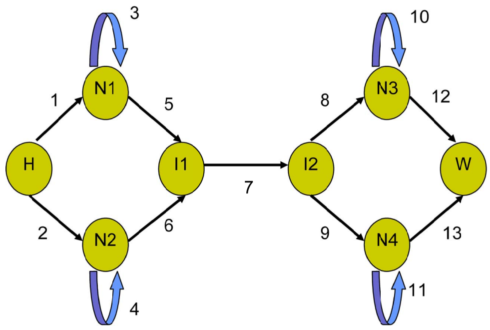


Figure 1. Example Network - Double-Diamond Network

On the other hand, if traffic dynamics is incorporated in the assignment model, we would obtain a solution where none of the individuals visit the non-work activity center in the R-diamond. In this case, there is no delay for any of the individuals and the total overall utility is 125 utils more than the previous case. The reason for the difference in utilities is the limited capacity of link 7. Ignoring the traffic flow dynamics, could lead to sub-optimal assignment patterns. Therefore, it is important to consider transportation demand-supply dynamics.

Activity demand-supply dynamics also play a similar important role in individual decisions. Examples include activity centers with access time restrictions, social interaction activities that provide greater utility with increased participation and capacity restrictions in shopping mall check-out counters. Consider the example of capacity restrictions in shopping mall check-out counters: current models that ignore such an activity supply capacity restriction could over-estimate trip-chaining of shopping activity by commuters or under-estimate non-peak hour shopping trips. If in the *double-diamond* network example above, the non-work activity centers located in the B-diamond had the following modified utility specification: 100 utils if flow on arc is less than or equal to 15 individuals, 75 otherwise; then, the corresponding destination choice and traffic assignment model would result in 15 individuals choosing to participate in the non-work activity in B-diamond while 10-individuals choose the R-diamond. ATNs can model the above described as well as several other activity demand-supply dynamics.

3 Conceptual Framework

We present the overall conceptual framework in this section. A similar conceptual framework for the general transportation planning problem was presented by Florian et al (1988). The framework presented here builds on the work by Florian et al (1988) and includes activity characteristics in addition to travel characteristics.

3.1 Definitions and Notation

h: index for household.
i_h: index for individuals in household h.
  $i_h \in 1, 2, \ldots, I_h$.
$G = \{\nu, \alpha\}$ is the activity-travel network, where ν is the set of nodes and α is the set of arcs.
  $\alpha \ni \{\alpha^T, \alpha^A\}$ correspond to the set of travel and activity arcs.
A_{i_h}: Set of activities individual i_h participates in.
$A^{trav}_{i_h}$: Set of travel activities for individual i_h.
$A_\bullet$: Set of activities for all individuals residing in node.

The elements of the above sets are characterized by attributes that denote their 'state'. We represent the set of characteristics as $\Omega[.]$.

$\Omega_\nu[X_\nu]$: Representing accessibility measures for different activities at node ν.

$\Omega_{\alpha^T}[(m,n),(f,TT(f))]$: Representing source and sink node, flow, and travel-time of travel arc α^T. The source and sink node are shown together because they represent known characteristics while flow and travel-time have to be solved for.

$\Omega_{\alpha^A}[(n,\delta),(U,f)]$: Activity-center node, duration, utility of traversing the arc, and flow in arc.

$\Omega_{A_{i_h}}[t_s,\delta,n]$: Activity start time, duration, and location node.

$\Omega_{A_{i_h}^{trav}}[t_s, \delta, o, d, \mu, \rho]$: Travel start time, duration, origin, destination, mode, and route.

$\Omega_h[.]$: Characteristics of the household h such as type of household, number of vehicles.

$\Omega_{i_h}[.]$: Characteristics of individual such as age, gender, employment status.

3.2 Relationships: ATN Framework

We use two types of functional relationships, Φ and Ψ, to capture the various complex relationships between the above variables. Φ functions are direct functional maps from $\Re^{mn}$ to $\Re^m$ (for example, regression equations), while Ψ functions represent more complex relationships such as a fixed-point mapping.

Several different frameworks arise based on the relationship assumptions among the above sets and their characteristics. The set of relationships below represent the framework adopted in this paper.

$$A_{i_h} = \Phi(\Omega_h, \Omega_{i_h}, \Omega_\nu, \hat{\Omega}_\alpha) \ \ \forall\ i_h \tag{1}$$

$$\{\Omega_{A_\bullet}, \Omega_{A_\bullet^{trav}}\} = \Psi_1(\Omega_{\alpha^T}, \Omega_{\alpha^A}, \Omega_\nu) \ \ \forall\ A \tag{2}$$

$$\{\Omega_{\alpha^T}[f, TT(f)], \Omega_{\alpha^A}[U, f]\} = \Psi_2(\Omega_{A_\bullet}, \Omega_{A_\bullet^{trav}}) \ \ \forall\ \alpha^T \text{and } \alpha^A \tag{3}$$

The reader may note that the Φ function is at an individual level while the Ψ functions are at a network or zonal level. Also, the Φ function is similar to disaggregate demand models while Ψ_2 is similar to an aggregate network assignment model. Given the complexity of the Ψ functions they are not modeled at an individual or disaggregate level.

Φ determines the set of activities that an individual participates in. Among other factors, this could depend on household and individual characteristics, activity center location and accessibility characteristics, transportation and activity supply characteristics, and also the set of fixed activities the individual participates in. The reader may note that the Φ function includes estimates of transportation and activity supply characteristics denoted as $\hat{\Omega}_\alpha$. In this paper, we assume Φ, the set of activities that an individual participates in, as known; the focus of this paper is on the two Ψ-functions only.

The Ψ functions represent complex relationships between arc (both travel and activity arc) characteristics and characteristics of activities. Ψ_1 determines the characteristics such as start time, duration and location for the set of activities and corresponding travel of all individuals in every node. They are assumed to depend on activity and travel supply characteristics represented by duration/traversal time, flow, location, and on activity-center accessibility characteristics Ω_ν. Ψ_2, on the other hand, maps a given set of activity and travel characteristics to a set of flows and corresponding travel times and utilities on arcs. The two relationships Ψ_1 and Ψ_2 together represent the fixed-point problem shown below.

$$\{\Omega_{\alpha^T}[f, TT(f)], \Omega_{\alpha^A}[U, f]\} = \Psi_2(\Psi_1(\Omega_{\alpha^T}, \Omega_{\alpha^A}, \Omega_\nu)) \tag{4}$$

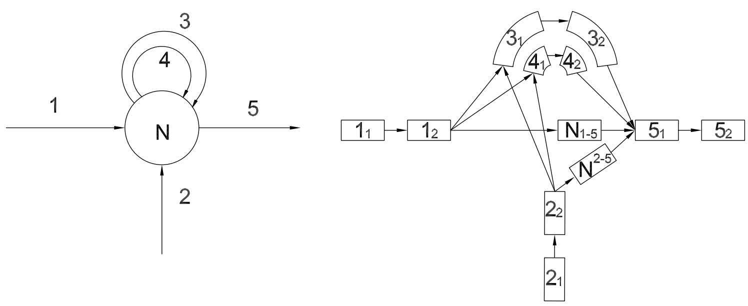
4 Operational Framework

Two critical issues to operationalize the ATN framework are flow propagation dynamics and utility function specification. We discuss their implementation details below.

4.1 Dynamics of Flow Propagation

Traffic flow has been modeled at different levels in the past. The most realistic models are disaggregate microsimulation models (Gartner et al, 2001) where behavior of each vehicle on the network is modeled explicitly. On the other hand, macroscopic models (Gartner et al, 2001) describe traffic flow based on relationships between speed, flow, and density. Though microscopic models are more accurate they require greater computation time and lack analytical solutions.

Macroscopic models, on the other hand, can be modeled as side constraints to provide approximate, quick solutions and are more suitable for analytical DTA models. Macroscopic models can be further divided into exit flow models, point queue models, and physical queue models. In this study, we use a network level simulation adaptation of the cell transmission model (CTM) (Daganzo, 1994, 1995; Ziliaskopoulos, 2000). The CTM is capable of capturing the effect of spillbacks (physical queue) and shock wave propagation (two-regime flow). Also, the fixed-point problem in equation 4 is formulated as a variational inequality (VI) problem. Existing VI solution techniques are based on heuristic searches and require several iterations of network loading step. Therefore, embedding a microsimulation model would require multiple runs of a computationally intensive model and could significantly increase the running time of any algorithm. We present the details of the CTM below.



Arcs 1 and 2 are incoming (into Node N) travel arcs.
Arcs 3 and 4 are activity arcs. Arc 5 is an outgoing travel arc.

The equivalent cell-based representation of arcs is shown on the right.
1_1 and 1_2 are two cells representing arc 1.

Figure 2. Cell-based Representation in ATNs

We assume the activity-travel network to be divided into a series of inter-linked cells. Cells represent a segment of a travel link or an activity location. Unlike (Daganzo (1994), Daganzo (1995)) we assume variable cell lengths. As mentioned in Daganzo (1995) this implies a trade-off between computational resource requirements and the level of accuracy of the CTM model to the classic LWR model - shorter cells can more closely replicate the LWR model but may demand more computational resources. Allowing for variable cell lengths is straight-forward in simulation adaptations of the CTM. Activity cells do not have a physical length; traversal time of activity cells are determined based on the duration of activity participation information contained in the route chosen for travel. The links between cells do not have any physical significance. An example of the cell transmission model representation of activities at a node is shown in Figure 2.

Notation:

Let,

$x_t^{i,r}$ is the number of vehicles following route r in cell i that entered the cell at $(t-t_f^i)^{th}$ time interval or earlier. In the case of activity cells, this represents the number of vehicles that have 'resided' in cell i for a time period equal to the duration of activity participation.

$x_t^i = \sum_{\forall r} x_t^{i,r}$

X_t^i: the total number of vehicles in cell i at time t.

$y_t^{i,r}$: flow on route r, out of cell i at time t.

$y_t^i = \sum_{\forall r} y_t^{i,r}$

N^i: Number of vehicles that can be accommodated at jam density for cell i.

Q^i: Maximum flow capacity out of cell i.

We assume N and Q are time invariant and therefore drop the time subscripts in their representation.

In the discussion below, cells j and k are assumed to be immediately downstream of cell i and cells g and h are immediate upstream of cell i.

Finally, P^g represents the fraction of flow from cell g that enters the downstream merge cell, i. The sum over all such fractions is one (in our illustration we assume two cells - g and h - merge into cell i. Therefore $P^g + P^h = 1$).

Flow propagation is achieved by repeatedly solving three sets of equations - first set of equations determine the outflow from a cell (y) between time-step *t-1* to t, second set of equations determine the individual route break-ups and the final set determine the current cell occupancy ($x_t^{i,r}$) based on past occupancy, inflows, and outflows.

y_t^i is determined from the following equations:

For activity cells: $y_t^i = x_t^i$.

For ordinary travel cells: $y_t^i = \min(x_{t-1}^i, Q^i, N^j - X_{t-1}^j)$.

For cells that merge into a single cell, several cases arise. We deal with each below:

Case 1: If $\min(x_{t-1}^g, Q^g) + \min(x_{t-1}^h, Q^h) > (N^i - X_{t-1}^i)$

Case 1a: If $\min(x_{t-1}^g, Q^g) > P^g(N^i - X_{t-1}^i)$ and $\min(x_{t-1}^h, Q^h) > P^h(N^i - X_{t-1}^i)$, then $y_t^g = P^g(N^i - X_{t-1}^i)$ and $y_t^h = P^h(N^i - X_{t-1}^i)$.

Case 1b: Else If $\min(x_{t-1}^g, Q^g) \leq P^g(N^i - X_{t-1}^i)$, then $y_t^g = \min(x_{t-1}^g, Q^g)$ and $y_t^h = (N^i - X_{t-1}^i) - y_t^g$.

Case 1c: Else, $y_t^h = \min(x_{t-1}^h, Q^h)$ and $y_t^g = (N^i - X_{t-1}^i) - y_t^h$.

Case 2: Else, $y_t^g = \min(x_{t-1}^g, Q^g)$ and $y_t^h = \min(x_{t-1}^h, Q^h)$.

For diverge cells, x_t^i is split into two parts x_t^{i,r_j} and x_t^{i,r_k} such that $x_t^{i,r_j}(x_t^{i,r_k})$ contains all vehicles that take cell j (k) next. This is determined based on the next cell in route r. The outflow into each of the two diverge links may be determined similar to an ordinary cell with route specific cell occupancies x_t^{i,r_j} and x_t^{i,r_k} instead of x_t^i.

The second set of equations determine the flow on each route r: $y_t^{i,r} = \frac{y_t^i}{x_{t-1}^i} x_{t-1}^{i,r}$.

The third step of determining current occupancy follows from $x_t^{i,r} = x_{t-1}^{i,r} + y_t^{j,r} - y_t^{i,r}$.

The reader is referred to Lo and Szeto (2002) for a detailed discussion on obtaining average travel times from the CTM simulation. An additional step required in the current model is to deduct activity participation durations from the computed travel times.

4.2 Utility Function Specification

The next critical step in the ATN framework is the utility function specification. The focus of the present paper is not on estimating utility function form or parameters. We assume reasonable functional forms and parameter values to illustrate the ATN framework. However, accurate estimation of utility function form and parameters is an important issue that needs further investigation in the future.
Let,
A^c be the set of all possible activity combinations.

R_{od}^a : Set of routes from origin o to destination d containing activity arcs α^A such that they traverse all activities in activity combination $a \in A^c$. r is a route that belongs to the set R_{od}^a. Each route r represents a set of travel and activity arcs. Therefore choosing a route r, results in the choice of activity location, duration, time of participation and travel route.

$U_{od}^{a,r}$ denotes utility derived by individuals departing from o and reaching d, participating in activity chain $a \in A^c$ using route r.

$h_{od}^{a,r}$: Path flow from o to d, participating in activity chain combination $a \in A^c$ using route r.

The temporal dimension in dynamic traffic assignment models (such as departure or arrival time index) is not associated with the above definitions since all individuals are always traveling on the network or participating in an activity.

Similar to Lam and Huang (2003), we assume an additive specification for the above utility expression.

$$U_{od}^{a,r} = U^a(r) - U^{trav}(r) \tag{5}$$

where, $U^a(r)$ is the utility derived from participating in activity combination $a \in A^c$ and is a function of route r. $U^a(r)$ can be represented as the sum of utilities derived from traversing each activity arc α_a in route r.

$$U^a(r) = \sum_{\forall \alpha^A \in r} U_\alpha(r, f) \tag{6}$$

where, f is the flow in activity link. In general, utility derived from activity participation may be assumed to be a function of type and duration of activity, time of participation, location of activity with respect to the origin/destination of flow on route r, and the total flow on activity link α^A.

$U^{trav}(r) = \beta * TT(r)$ is the disutility from travel on route r. where, β is a parameter to convert travel-time into utility units and $TT(r)$ is the total travel time on route r.

5 Mathematical Formulation of ATNs

5.1 Dynamic User Equilibrium Conditions

We can now express the DUE conditions as follows:

$$U_{od}^{a,r} = \begin{cases} = \overline{U}_{od}^a \text{ if } h_{od}^{a,r} > 0 \\ \leq \overline{U}_{od}^a \text{ if } h_{od}^{a,r} = 0 \end{cases} \quad \forall \text{o,d,} a \in A^c, \text{and r}(: r \in R_{od}^a) \tag{7}$$

Subject to the condition that flow on network should satisfy demand. This is expressed as:

$$\sum_{\forall r \in R_{od}^a} h_{od}^{a,r} = \sum_{\forall i_h \in (o,d)} \zeta_{i_h}^a \quad \forall a \in A^c, o, d \tag{8}$$

where, $\zeta_{i_h}^a = \begin{cases} 1 \ldots \text{ if activity combination } a \in A_{i_h} \\ 0 \ldots \text{ otherwise} \end{cases}$

$\overline{U}_{od}^a$ is the maximum utility derived by individuals departing from o and reaching d, participating in activity combination $a \in A^c$ using route r.

DUE conditions, however, are not always satisfied in capacitated networks (Szeto and Lo, 2006). Discontinuities in travel time or utility functions could result in non-existence of solutions. These discontinuities could arise from time discretization or due to capacity restrictions in the network. In capacitated networks it is possible that packets of flow are broken because of the lack of available capacity downstream. Any discrete-time model in capacitated networks exposes itself to the above drawback. Further study is required to understand the properties of DUE in discrete-time capacitated network models.

5.2 Equivalent variational inequality formulation

The above DUE conditions can now be formulated as an equivalent VI problem.

$$\sum_{\forall a \in A^c} (\mathbf{h}^a - \hat{\mathbf{h}}^a)^T U^a(\mathbf{h}) \geq 0 \quad \forall \mathbf{h}^a \in H^a \text{ and } \forall a \in A^c \tag{9}$$

where,
H^a is the set of feasible route flows traversing all activities in activity combination a, given by (7),
$\mathbf{h}^a$ is the vector of route flows $\in H^a$,
$\hat{\mathbf{h}}^a$ is the vector of route flows that satisfy the DUE condition in equation 6, and
U^a is a vector whose each element is given by $U^{a,r}_{od} - \overline{U}^a_{od}$.

5.3 Solution Approach

The utility derived from traversing the activity-travel sequence represented by route r, expressed as the sum of utility derived from participating in activities and the disutility from travel, is assumed to be a monotone decreasing function of flow on route r. Therefore, a route-swapping algorithm (Lam and Huang, 2003; Szeto and Lo, 2006; Nagurney and Zhang, 1997) is adopted to obtain solutions to the VI problem shown in (8). The detailed algorithm is presented below:

Step 0: Initialize. Set iteration counter $i = 0$.
Choose an initial feasible vector of flows $\mathbf{h}(i)$.

Step 1: Computation. Load flow $\mathbf{h}(i)$ and compute travel times $\mathrm{TT}(r)$ using the Cell-based transmission model.

Compute utilities $U^{a,r}_{od}$ using (5) $\forall\ r,\ a,\ o,\ d$.

Set $\overline{U}^a_{od} = \max_{\forall r \in R^a_{od}} U^{a,r}_{od} \quad \forall\ a,\ o,\ d.$

Step 2: Update flows. Set $\hat{R}^a_{od} = r \in R^a_{od} : U^{a,r}_{od} = \overline{U}^a_{od}$.
For ever activity combination $a \in A^c$,

$$h^{a,r}_{od}(i+1) = \max(0, h^{a,r}_{od}(i) + \rho h^{a,r}_{od}(i)(U^{a,r}_{od} - \overline{U}^a_{od}))\ \forall r \in R^a_{od} \backslash \hat{R}^a_{od}$$

$$\Sigma^a_{od} = \sum_{\forall r \in R^a_{od} \backslash \hat{R}^a_{od}} (h^{a,r}_{od}(i) - h^{a,r}_{od}(i+1)) \quad \forall\ a,\ o,\ d.$$

$$h^{a,r}_{od}(i+1) = h^{a,r}_{od}(i) + \frac{\Sigma^a_{od}}{|\hat{R}^a_{od}|} \quad \forall r \in \hat{R}^a_{od} \quad \forall\ \ r,\ a,\ o,\ d.$$

ρ is a scale parameter.

Step 3: Check for convergence. Compute $\pi = \sum_{\forall r,a,o,d} (U^{a,r}_{od} - \overline{U}^{a}_{od})\, h^{a,r}_{od}$ and $\hat{\pi} = \sum_{\forall r,a,o,d} \overline{U}^{a}_{od}\, h^{a,r}_{od}$.
If $\frac{\pi}{\hat{\pi}} < \epsilon$ then terminate. ϵ is a convergence tolerance value.
else, i = i + 1; Go to Step 1.

Nagurney and Zhang (1997) use the route-swapping algorithm (referred to as Euler's method) for the static traffic assignment problem. They show that the Euler's method converges only when the link costs are strictly monotone increasing. However when implementing the algorithm for path based formulations they reported that the algorithm did not converge in their limited trials. Other studies (Lam and Huang, 2003; Szeto and Lo, 2006) also report lack of smooth convergence in their implementations of the algorithm. Therefore, the route swapping algorithm has convergence issues when implemented in path-based formulations. In the numerical trials we test different scenarios and report under what conditions the route swapping algorithm appears to provide consistent DUE solutions to a test problem.

Two important components in the above algorithm are the convergence expression and the scale parameter ρ. Traditionally, the literature has utilized a convergence check based on flow changes between two iterations (Nagurney and Zhang, 1997; Abdelghany et al, 2003). However, since path flows are not necessarily unique (assuming the solution exists), using flow to determine convergence could lead to infinite loops in the algorithm. A better convergence measure is the utility difference of used paths in successive iterations. However, a direct comparison of utilities in used paths could lead to the algorithm converging to a non-equilibrium solution. The expression used above overcomes this problem by comparing the utility on all used paths to the maximum possible utility. This ensures that the algorithm converges only when all used paths have an utility that is close the maximum possible utility at equilibrium. If there are no equilibrium solutions, the algorithm will not terminate. A reasonable upper limit on the number of iterations is required to ensure the algorithm does not loop infinitely. Consequently, if the algorithm terminates after reaching the maximum number of iterations, the solution must be checked to see if it has converged to an equilibrium solution or not.

Nagurney and Zhang (1997) provide conditions for the scale parameter ρ under which the algorithm converges to an equilibrium solution. Lam and Huang (2003) show that these conditions allow for local stability if the cost functions are not strictly monotonic. However, in discrete time capacitated networks such as the one dealt with in this paper, the utility functions are likely to be discontinuous with sudden jumps and falls. This adds to the complexity of the algorithm convergence and no theoretical properties for the value of the scale parameter ρ exist. This problem is also reported in Szeto and Lo (2006). We test different values for the scale parameter and draw limited insights.

6 Results from an Example Network

We demonstrate the application of the ATN framework and the proposed solution algorithm on an example network. The example network considered is the double-diamond network presented earlier. The equivalent cell-based representation of the network is shown in figure 3. Free-flow traversal time, maximum flow capacity, and number of vehicles at jam density for square (rectangular) cells are 1 minute, 1000 vehicles/minute, and 3500 vehicles (3 minute, 1000 vehicles/minute, and 10500 vehicles).

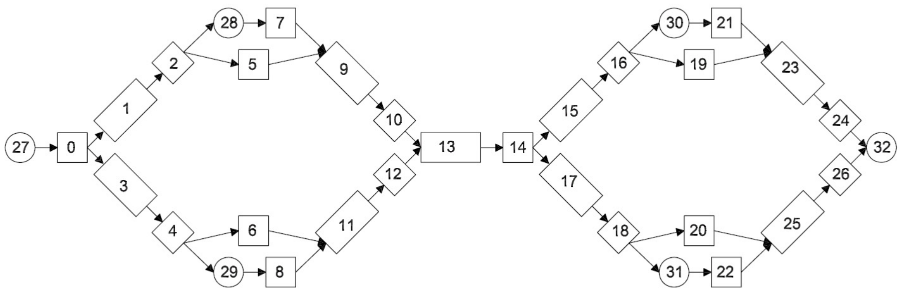


Figure 3. CTM Representation of Double-Diamond Network

There are two possible activity chains - home to work (H-W) and home to non-work to work (H-NW-W). All individuals depart from home; departure times are set at every fifth minute starting from (and including) 7:00 AM. The preferred work arrival time is 8:00 AM. The possible non-work activity durations are 5, 10, 15, 20, 25 and 30 minutes. Since free-flow travel time is 25 minutes, 7:35 AM (7:30 AM) is assumed to be the latest departure time for individuals participating in H-W (H-NW-W) activity chain. There are 4 (8) travel route options available for activity chain H-W (H-NW-W). Therefore, for the H-W activity chain there are (4x8=) 32 route options numbered from zero to thirty one [1], while there are (8x27=) 216 route options for the H-NW-W activity chain [2] numbered from thirty two to two hundred and forty seven. Three demand levels are analyzed. Low, medium, and high demand representing 750, 3750, and 7500 individuals participating in each activity chain combination are considered.

The utility profiles for the different activities are presented in Figure 4. Home stay is rewarded with 100 utils/min. The utility derived from non-work activity participation are identical for all four locations; they depend on duration of participation only. Starting at zero utils for zero minutes duration, the utility derived from every additional minute increases linearly to a maximum of 125 utils/min for duration of 15 minutes and then drops linearly to 0 utils/min at 30 minutes of activity participation. The utility is assumed to be independent of time of participation or flow on activity arc. The preferred arrival time at work is 8:00 AM. Early arrivals are penalized at the rate of -50 utils/min and late arrivals are penalized at -150 utils/min. Travel disutility is -100 utils/min of travel.

Three different scale parameter ρ values were tested. ρ was first assigned to an initial value (*1/n*) and then progressively reduced to 1, 1/2, 1/3, 1/4... of the initial value. Each value was held constant for n iterations. For example when n = 10, the initial value of ρ is set to 0.1 for the first 10 iterations, then reduced to 0.05 (= 0.1/2)

[1] Includes departure time options 7:00, 7:05, ... 7:35 and 4 travel routes

[2] Including 4 locations, 2 travel routes, and 6 non-work activity duration options for individuals departing at 7:00 AM and 7:05 AM, 5 for 7:10 AM departures, 4 for 7:15 AM departures and so on - totaling to 27 activity duration combinations.

for iterations 11 through 20, then reduced to 0.033 (= 0.1/3) for iterations 21 through 30 and so on. Three different values of n (100, 1000, and 10000) were tested.

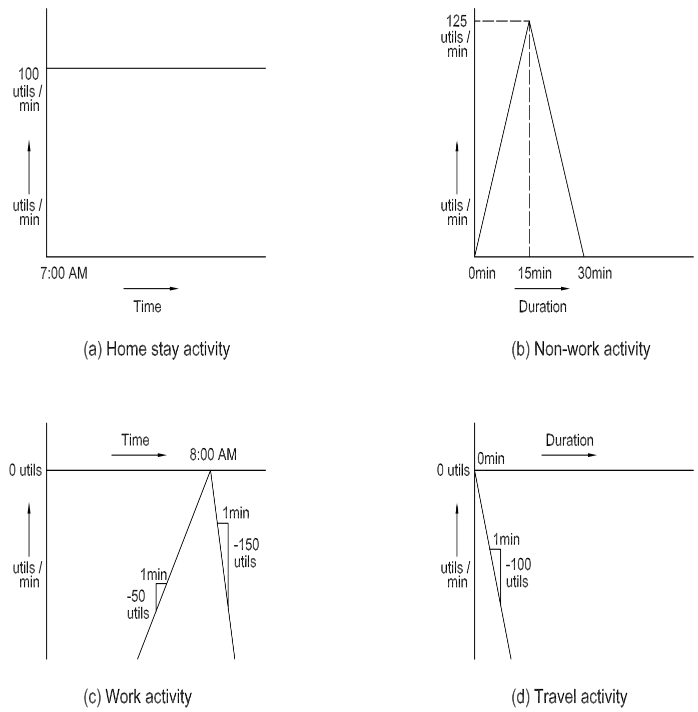
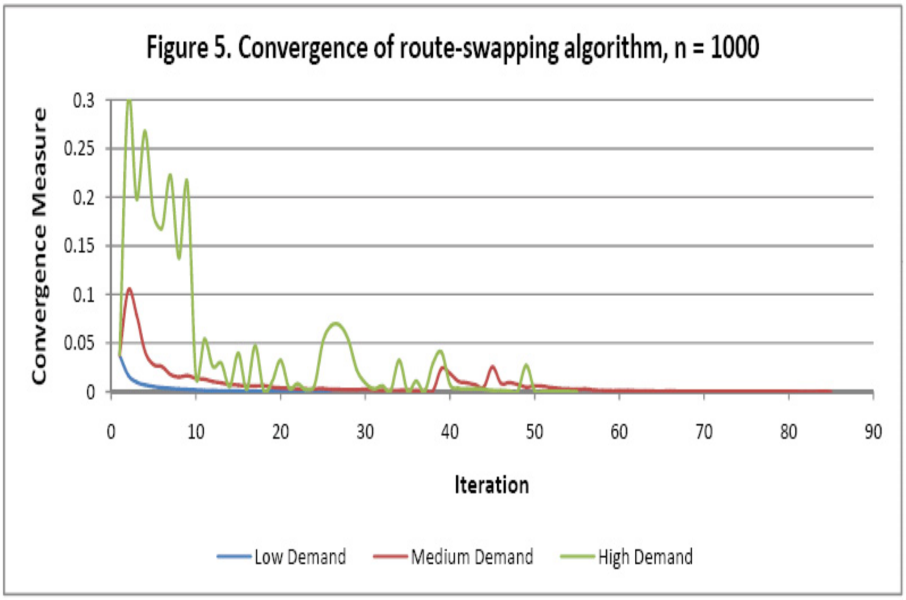
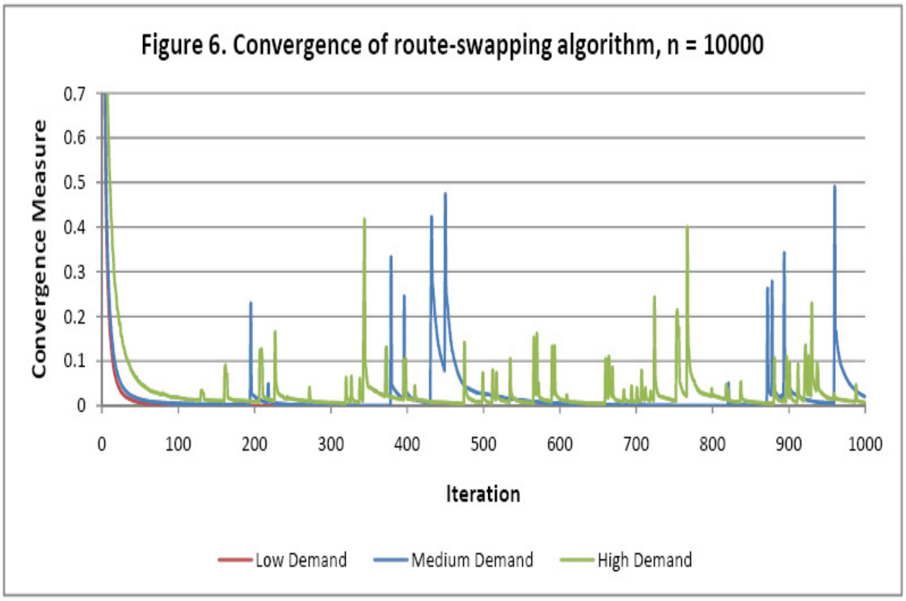


Figure 4. Utility Profiles





In terms of the overall results, the high demand case did not converge. It appears that there is no equilibrium solution for the high demand case. The inverse of the initial value of the scale parameter, $n = 100$ converged immediately (to a non-equilibrium solution) and have not been included in the results presented below.

The convergence measure for $n = 1000$ and $n = 10000$ are presented in Figures 5 and 6. As can be seen from the figures, the low demand case converges to an equilibrium very smoothly. There are a few spikes in the medium demand case (for both values of n), and even more spikes for the high demand case. Further, the rate of convergence is faster for the lower value of n (higher value of scale parameter ρ). This is consistent with the result in Szeto and Lo (2006).

As mentioned earlier the high demand case did not have any equilibrium solution. However, in general the algorithm progressed such that the maximum flow was loaded on to the route with maximum utility. This result was observed only for $n = 10000$. Even though the rate of convergence is faster for $n = 1000$, the solution obtained for the high demand case was poor. That is the maximum flow was not among routes with maximum utilities.

The equilibrium flows and corresponding average utilities for both H-W and H-NW-W activity chains for all three demand levels are presented in Table 1. The solutions correspond to converged values for the low and medium demand cases and 100,000 iterations for the high demand case. The value of n used here is 10000. The low and medium demand case have converged to an equilibrium solution since almost 99.9% of the flows are in routes that provide maximum utility. In the high demand case, however, the utility at equilibrium for almost all used paths in the H-W activity chain and for 98.3% of flow for the H-NW-W activity chain are equal. About 1.7% of the flows in the H-NW-W activity combination have utilities lesser than the maximum possible utility - this could be because of the capacity restrictions not allowing these flows to shift to a more attractive path without adversely affecting the utilities of others. The presence of unequal flows even among symmeteric paths - 31, 7, 23 and 15 for example - (notwithstanding approximations arising from sequential flow propagation steps) indicates the possibility of non-uniqueness of solutions in terms of path flows in DUE.

An intuitive result obtained in the example in Section 2 is also corroborated here. The intuition is that individuals would prefer participating in the non-work activity at the location in B-diamond instead of R-diamond to avoid queuing at the bottleneck link joining the two diamonds. We observe this in Table 1b. The flows on routes shown in the table accounts for 98.3% of equilibrium flow and all individuals contributing to this flow prefer the non-work activity center in B-diamond (cells 30 and 31 shown in Figure 3). In terms of durations, individuals in the H-W activity chain prefer departing as late as possible (7:35 AM) while few depart at 7:30 AM. For H-NW-W activity chain, 7:30 AM departure and a non-work activity duration of 5 minutes is the most preferred.

In summary, the route-swapping algorithm performed reasonably well. Even for high demand cases when there is no equilibrium solution the algorithm approaches a 'good' solution that ensures more flow on higher utility paths. However as demand increases the convergence is not smooth - there are several spikes as seen in Figures 5 and 6. A reason for the spikes may be because of the discrete time capacitated network considered here - a small shift in flow to some paths may result in substantial delays and disutilities. Also, the convergence value was found to depend on the ρ parameter though clear patterns were not obtained. These remain important issues for future studies.

Table 1a. Equilibrium flows for activity-chain H-W

Route Id	Low Demand		Medium Demand		High Demand		Home Stay Duration
	Flow	Utility	Flow	Utility	Flow	Utility	
31	197.72	1050	802.90	796.71	1861.90	243.49	35 min
7	172.86	1050	1039.34	796.71	1759.29	244.93	35 min
23	153.16	1050	1072.82	796.71	1768.86	244.38	35 min
15	226.21	1050	834.94	796.72	1712.35	244.04	35 min
6	0	300	0	300	89.74	239.43	30 min
22	0	300	0	300	115.47	239.42	30 min
30	0	300	0	300	98.35	239.42	30 min
14	0	300	0	300	94.04	239.43	30 min
Other routes (24 nos.)	0	< 300	0	< 300	0	≤ -450	≤ 25 mins

Table 1b. Equilibrium flows for activity-chain H-NW-W

Route Id	Low Demand		Med Demand		High Demand		Home Stay Duration	NW Activity Duration	NW Activity Location
	Flow	Utility	Flow	Utility	Flow	Utility			
220	96.3	654.2	801.6	472.7	783.4	115.2	30 min	5 min	31
139	191.2	654.2	611.5	472.6	756.3	116.2	30 min	5 min	30
112	148.1	654.2	708.8	472.7	711.4	116.3	30 min	5 min	30
247	64.1	654.2	900.7	472.6	705.9	115.2	30 min	5 min	31
214	0	508.3	134.6	472.7	413.2	116.1	15 min	20 min	31
133	0	508.3	182.9	472.7	406.5	116.4	15 min	20 min	30
241	0	508.3	227.2	472.7	402.3	116.1	15 min	20 min	31
106	0	508.3	179.0	472.7	455.7	116.4	15 min	20 min	30
136	0	487.5	0.1	451.9	349.2	116.5	20 min	15 min	30
109	0	487.5	0.1	451.9	345.2	116.5	20 min	15 min	30
244	0	487.5	0.1	451.9	346.8	116.3	20 min	15 min	31
217	0	487.5	0.1	451.9	394.7	116.3	20 min	15 min	31
138	0	466.7	0	466.7	313.2	116.3	25 min	10 min	30
219	0	466.7	0	466.7	313.1	116.2	25 min	10 min	31
246	0	466.7	0	466.7	313.1	116.2	25 min	10 min	31
111	0	466.7	0	466.7	317.5	116.3	25 min	10 min	30
193	77.3	654.2	0.4	419.8	11.8	66.6	30 min	5 min	29
85	74.0	654.2	0.8	391.6	11.2	46.5	30 min	5 min	29
166	50.0	654.2	0.4	419.8	13.4	66.4	30 min	5 min	28
58	48.3	654.2	0.8	391.6	10.88	46.3	30 min	5 min	28
Other 196 nos.	= 0%	≤ 300	= 0%	≤ 466.7	< 1.7%	≤ 69.8	Varies	Varies	Varies

7 Summary and Further Work

In this paper, an integrated formulation to obtain equilibrium solutions across multiple dimensions of travel choice is presented. The formulation is based on a Supernetwork representation referred to as Activity-Travel Network (ATN) representation. In ATN representation, nodes are activity centers that are joined by travel links. Activities are represented by arcs that both originate and terminate in the same node (activity centers). An activity-travel sequence for an individual can be represented as a 'route' that includes both travel and activity arcs. The 'route' choice in an ATN results in simultaneous determination of activity location, time of participation, duration, and route choice decisions.

The proposed integrated framework allows (a) to capture activity demand-supply dynamics in addition to transportation demand-supply dynamics, and (b) to obtain a consistent equilibrium solution across all dimensions of choice. A rigorous mathematical and operational framework for ATNs based on dynamic user equilibrium behavior with an embedded cell-based transmission traffic flow model was presented. The equivalent variational inequality problem was obtained. A solution method based on route-swapping algorithm is proposed and demonstrated on an example network.

Several open issues merit further investigation: first, we need to derive the properties such as solution existence and uniqueness of the variational inequality problem. Second, numerical or analytical results on convergence properties of solution algorithms need to be developed. This would depend on the utility function specification and the traffic flow dynamic model among other factors. Third, more sophisticated representation for the utility function and the activity-travel choice mechanism can be explored. Finally, the solution algorithm presented here did not converge smoothly for higher demand values (more congested cases). While this can be improved by adopting a finer resolution of time discretization, it leads to increase in the number of route alternatives. Algorithms that obviate the need for route enumeration can solve the problem significantly faster. Faster solution algorithms will allow the Activity-Travel Network framework to be adopted in real-time traffic management applications even in large scale networks.

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