University Transportation Research Center - Region 2

Final Report

Estimating Multi-class Truck Origin-Destination Flows Through Data Fusion from Multiple Sources

Performing Organization: Cornell University

November 2013

Sponsor:
University Transportation Research Center - Region 2

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The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

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The research program objectives are (1) to develop a theme based transportation research program that is responsive to the needs of regional transportation organizations and stakeholders, and (2) to conduct that program in cooperation with the partners. The program includes both studies that are identified with research partners of projects targeted to the theme, and targeted, short-term projects. The program develops competitive proposals, which are evaluated to insure the mostresponsive UTRC team conducts the work. The research program is responsive to the UTRC theme: "Planning and Managing Regional Transportation Systems in a Changing World." The complex transportation system of transit and infrastructure, and the rapidly changing environment impacts the nation's largest city and metropolitan area. The New York/New Jersey Metropolitan has over 19 million people, 600,000 businesses and 9 million workers. The Region's intermodal and multimodal systems must serve all customers and stakeholders within the region and globally.Under the current grant, the new research projects and the ongoing research projects concentrate the program efforts on the categories of Transportation Systems Performance and Information Infrastructure to provide needed services to the New Jersey Department of Transportation, New York City Department of Transportation, New York Metropolitan Transportation Council , New York State Department of Transportation, and the New York State Energy and Research Development Authorityand others, all while enhancing the center's theme.

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The modern professional must combine the technical skills of engineering and planning with knowledge of economics, environmental science, management, finance, and law as well as negotiation skills, psychology and sociology. And, she/he must be computer literate, wired to the web, and knowledgeable about advances in information technology. UTRC's education and training efforts provide a multidisciplinary program of course work and experiential learning to train students and provide advanced training or retraining of practitioners to plan and manage regional transportation systems. UTRC must meet the need to educate the undergraduate and graduate student with a foundation of transportation fundamentals that allows for solving complex problems in a world much more dynamic than even a decade ago. Simultaneously, the demand for continuing education is growing – either because of professional license requirements or because the workplace demands it – and provides the opportunity to combine State of Practice education with tailored ways of delivering content.

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UTRC-RF Project No: 49111-22-23

Project Date: November 2013

Project Title: Estimating Multi-class Truck Origin-Destination Flows Through Data Fusion from Multiple Source

Project's Website:
http://www.utrc2.org/research/projects/multi-class-truck-origin-destination

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Sponsor:
University Transportation Research Center - Region 2, A Regional University Transportation Center sponsored by the U.S. Department of Transportation's Research and Innovative Technology Administration

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Disclaimer

Acknowledgement

Research gift support from the University Activities Committee at Xerox Corporation is gratefully acknowledged. This gift, given for support of transportation systems research in the School of Civil & Environmental Engineering at Cornell, has been instrumental in furthering the work reported here. The active interest and involvement of Marina Tharayil, George Gibson and John Handley at Xerox has been of great help and is deeply appreciated.

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.

4. Title and Subtitle

Multiclass Origin-Destination Estimation Using Multiple Data Types

5. Report Date

November, 2013

6. Performing Organization Code

7. Author(s)

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8. Performing Organization Report No.

9. Performing Organization Name and Address

School of Civil & Environmental Engineering
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10. Work Unit No.

11. Contract or Grant No.

49111-22-23

12. Sponsoring Agency Name and Address

University Transportation Research Center
City College of New York-Marshak 910
160 Convent Avenue
New York, NY 10031

13. Type of Report and Period Covered

Final Report May 2011 – July 2013

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

There has long been interest in estimating origin-destination (O-D) matrices from link count (or other) data. For the most part, previous efforts have focused on creating O-D tables for passenger movements by automobile, using a single vehicle class. This report describes a formulation of the multiclass O-D estimation problem that is constructed to accept a variety of different types of data that relate O-D volumes to observed values. This includes traditional link count data, but also accommodates other types of data such as turning movements at intersections, partial path observations from individual vehicles, etc. This has particular interest for transportation planners interested in separating truck flows from auto flows for more effective overall system planning and management.

The O-D estimation process is constructed using a probit-based stochastic model for network loading and user equilibrium. One of the advantages of using this formulation is that it makes data on partial paths of vehicles useful in the O-D estimation process, and such information is becoming more available as a result of technological changes in vehicle sensing and data exchange.

The multiclass O-D estimation process is tested in two different environments – one a small network using three vehicle classes where the true O-D tables are known, and a second larger network (Rockland County, New York) using two vehicle classes where very limited data are available. The Rockland County test case demonstrates that the multiclass O-D estimation method developed in this research can successfully estimate trip tables in realistic networks, and thus has significant practical value. It also demonstrates that the addition of truck trip-end estimates based on economic activity data can change the estimated truck table significantly, and thus this data has considerable value as an augmentation to observed traffic data.

17. Key Words	18. Distribution Statement
Origin, Destination, Truck, Vehicle	

19. Security Classif (of this report)	20. Security Classif. (of this page)	21. No of Pages	22. Price
Unclassified	Unclassified		

Form DOT F 1700.7 (8-69)

EXECUTIVE SUMMARY

Since the mid-1970's, there has been interest in estimating origin-destination (O-D) matrices from link count (or other) data. For the most part, these efforts have focused on creating static O-D tables for passenger movements by automobile, using a single vehicle class. An important focus of this research is simultaneous estimation of O-D matrices for multiple vehicle classes that interact in their use of the network. One obvious distinction is between autos and various types of trucks in a road network, but the concept of vehicle classes also transfers to other situations (multiple train classes in a railroad network; multiple aircraft types using an airport, multiple vessel types in a lock-and-dam operation on a river, etc.). Different vehicle classes may use different amounts of available facility capacity, have different operating characteristics, etc.

Among transportation planners, interest in treating multiple vehicle classes separately is growing, especially in urban areas where truck flows are of increasing concern. It has been common in transportation planning studies to assume that truck flows are a constant (and generally small) fraction of automobile flows. However, this assumption ignores the substantial differences that may exist between the origins and destinations for trucks and those for automobiles. Trucks also impose different levels of pavement damage than cars, they have different emission characteristics, different accident patterns, and may be the subject of different types of flow controls. However, truck O-D estimation suffers from two fundamental challenges. First, it is inherently a multi-class problem because there are several different size classes of trucks, in addition to the differences between automobiles and trucks. Second, the available data on truck movements is very sparse, particularly data that separates different classes of trucks.

This report describes a formulation of the multiclass O-D estimation problem that is constructed to accept a variety of different types of data that relate O-D volumes to observed values. This includes traditional link count data, but also accommodates other types of data such as turning movements at intersections, partial path observations from individual vehicles, etc. The data may be either class-specific or may aggregate a subset of vehicle classes. A particular focus of this formulation is to allow better estimation of truck flows in urban networks, using various size classes of trucks, and separating them from estimation of automobile O-D patterns.

The O-D estimation process is constructed using a probit-based stochastic model for network loading and user equilibrium. Using a stochastic model of user equilibrium (SUE) allows a relaxation of the common assumption that all drivers have complete and accurate knowledge of link travel costs everywhere in the network. Although the probit model of SUE is more demanding computationally than a logit-based model, it has important conceptual advantages with respect to the assumption about the distribution of errors and in reflecting covariance among path alternatives that share common links. One of the advantages of using an SUE formulation is that it makes data on partial paths of vehicles useful in the O-D estimation process, and such information is becoming more available as a result of technological changes in vehicle sensing and data exchange.

An algorithm has been developed to implement the multiclass O-D estimation process and the various aspects of that algorithm are described in detail in Chapter 3. The algorithm has been tested in two different environments – one a small network using three vehicle classes where the true O-D tables are known, and a second larger network using two vehicle classes where very limited data are available. Because the true O-D tables are known in the first test case, a series of experiments is constructed to test the ability of the O-D estimation procedure to reproduce those tables from varying amounts of available observations. This set of experiments results in important conclusions regarding the amount and types of data necessary to support multiclass O-D estimation. In summary form, these conclusions are:

1) Without classified link counts, the estimation of truck O-D volumes is completely ineffective. Even if all sensors can differentiate between automobile and trucks (i.e. dual loop counters), the estimates of truck O-D volumes are nearly all incorrect. Using dual loop detectors to substitute for classification counts in estimating O-D tables with more than one truck class is not likely to prove useful.
2) When only single loop detectors exist, even with 100% link coverage, none of the multiclass O-D entries (automobile or truck) is estimated usefully. This is important because single loop counters are currently the dominant source of traffic counts in most urban areas. Moreover, the link coverage in practice is much less than 100%, and under these conditions, it is likely to be a futile exercise to attempt estimation of multiclass O-D tables.

3) Adding turning movement data to link counts is an effective way of increasing the quality of O-D estimates. When only total vehicle turning movements are available, the primary effect is on estimates of automobile (class 1) O-D volumes. If classification counts are available on links, addition of total turning movements at intersections improves the automobile estimation accuracy, although it provides little benefit for the truck classes.
4) When classification information is not available in the link count data, classified turning movements can improve estimation of truck O-D tables dramatically.
5) If vehicle classification counts are available from the link data, classified turning movements can be substituted by cheaper dual turning movements and have similar overall O-D estimation accuracy.
6) There is marked deterioration in the quality of the estimated O-D tables when the observed data contains errors of the magnitude that might normally be expected in practice. This is not unexpected, but this emphasizes two important aspects of the multiclass O-D estimation problem. First, it is important to reduce the errors in the observed data whenever possible. This may mean using better counting technology, or doing more filtering of the data before it is used. Second, in the presence of errors it is necessary to have far more observations to support O-D estimation than would be necessary in an error-free environment. This has implications for sensor location, which includes consideration of both how many sensors, and where they should be located.

The second test case is in a realistic setting, performed using Rockland County, New York, as the test network. The principal sources of data for this test case are average daily vehicle counts collected by the New York State Department of Transportation, and estimates of truck trip generation based on land use. The experiments with this test network are designed to answer two basic questions:

1) Does the methodology developed in this project scale reasonably to a much larger network with a much more complex structure and variety of facilities?
2) If basic count data is augmented by commonly available planning data (e.g., land use data) and generic truck trip generation estimates based on that data, how are the estimated O-D tables affected?

The Rockland County test case demonstrates that the multiclass O-D estimation method developed in this research can successfully estimate trip tables in realistic networks, and thus has significant practical value. It also demonstrates that the addition of truck trip-end estimates based on economic activity data can change the estimated truck table significantly, and thus this data has considerable value as an augmentation to observed traffic data.

Several avenues for further research are present. Two general directions of particular interest are sensor location and extension to dynamic O-D table estimation. Because local agencies are unlikely to have instrumentation deployed to observe flow volumes by vehicle class on all network links, an important concern is resource allocation. That is, with a limited budget for sensor acquisition and deployment, what locations and types of sensors should be selected to maximize the effectiveness of the data for O-D flow estimation across multiple vehicle classes? This sensor location problem is built upon the O-D estimation process developed here by adding consideration of the information content of different types of observations.

Extension of the methodology to allow dynamic O-D tables is also an important direction. It is clear that both auto flows and truck flows change during the day, and they may change fairly rapidly during some time periods. More effective flow control and traffic management across the day, and across different vehicle classes, requires better understanding of the temporal variation of O-D flows. Much progress has been made in recent years in developing dynamic traffic assignment methods for predicting network flows if the O-D flow rates are known. The challenge is to build methods for inferring the O-D flows from these methods and corresponding real-time observations of traffic flows. There are methods focused on auto traffic only, but little has been done yet in the domain of multiclass dynamic O-D estimation that might apply to both autos and trucks.

Chapter 1

INTRODUCTION

Since the mid-1970's, there has been interest in estimating origin-destination (O-D) matrices from link count (or other) data. For the most part, these efforts have focused on creating static O-D tables for passenger movements by automobile, using a single vehicle class. A sampling of some of the influential papers in this area includes the work of Robillard (1975), Turnquist and Gur (1979), Van Zuylen and Willumsen (1980), Carey, *et al*. (1981), Maher (1983), Cascetta (1984), Nguyen (1984), Spiess (1987), Cascetta and Nguyen (1988), Brenninger-Göthe, *et al*. (1989), Lam and Lo (1990), Yang, *et al*. (1992), Sherali, *et al*. (1994), Yang (1995) and Maher, *et al*. (2001).

Since the late 1990's, most of the effort on O-D matrix estimation has focused on estimating dynamic O-D tables for use in route guidance modeling and other applications of dynamic traffic assignment. These efforts also are aimed at a single vehicle class – automobiles. Some of the noteworthy literature in dynamic O-D estimation includes Van der Zijpp (1997), Ashok and Ben-Akiva (2000), Dixon and Rilett (2002), Zhou and Mahmassani (2005).

Because there is growing interest in planning for, and managing, truck movements in urban areas, there is a clear need for O-D estimation methods that treat multiple vehicle classes simultaneously. It has been common in transportation planning studies to assume that truck flows are a constant (and generally small) fraction of automobile flows. However, this assumption ignores the substantial differences that may exist between the origins and destinations for trucks and those for automobiles. Estimating O-D tables for trucks is of substantial interest to transportation planners because trucks impose different levels of pavement damage than cars, they have different emission characteristics, different accident patterns, and may be the subject of different types of flow controls. It is also important to recognize that there are several different size classes of trucks, whose O-D patterns are likely to be different from one another, and this emphasizes the need for general multi-class O-D estimation methods.

Part of the reason for the past focus on single-vehicle-class O-D estimation is that the primary data on which the estimation is assumed to depend are link counts of vehicles. These data are normally collected by loop detectors buried in the roadway. However, single-loop detectors, which are ubiquitous in practice, cannot reliably distinguish a truck from a car – they simply can count the total number of vehicles passing over the detector during a specified time interval. Because the traffic stream in most locations is dominated by automobiles, these total counts have been considered usable for single-class (automobile) O-D estimation. For multi-class O-D estimation, it is vital to have data that distinguishes among vehicle classes. Modern traffic sensing technology is providing increasing ability to classify vehicles as they are counted, as well as to create data that are more informative than simple link counts.

The O-D estimation method developed in this research is designed to include multiple vehicle classes whose O-D patterns are different, and to accommodate a wider variety of data types than just link counts. This creates an important new tool for transportation planners interested in truck movements (for several size classes) as well as automobile movements, and to take advantage of the increasing sophistication of traffic sensing technology.

Because much of this project is based on the ability to classify vehicles and to have sensors that can count vehicles by class, Chapter 2 focuses on vehicle classification and sensors. Chapter 3 describes the O-D estimation model formulation, and discusses the details of the solution process. Chapter 4 discusses results from a series of experiments on a small test network, and Chapter 5 describes application of the method to a larger realistic network (representing Rockland County, New York). Chapter 6 presents conclusions and directions for further work.

Chapter 2

VEHICLE CLASSIFICATION, SENSORS AND DATA TYPES

2.1 Vehicle Classification

Vehicle classification can be done in several different ways, but one of the standard classifications is that used by the Federal Highway Administration (FHWA). This classification system defines 13 vehicle classes, as listed in Table 2-1. Figure 2-1 illustrates the important truck classes. Trucks in classes 10-13 are seldom seen in urban areas, as they are mostly used for long-haul movements.

Table 2-1. FHWA vehicle classes.

Vehicle Class	Description
1	Motorcycles
2	Passenger cars
3	Two-axle, four-tire single unit trucks
4	Buses
5	Two-axle, six-tire single unit trucks
6	Three-axle single unit trucks
7	Four or more axle single unit trucks
8	Four or less axle single trailer trucks
9	Five-axle single trailer trucks
10	Six or more axle single trailer trucks
11	Five or less axle multi-trailer trucks
12	Six-axle multi-trailer trucks
13	Seven or more axle multi-trailer trucks

An alternative commercial vehicle classification with eight truck classes, based strictly on the gross vehicle weight, is also used quite widely. The first 6 classes are subdivisions of what are shown as types 3 and 5 in Figure 2-1. A “class 7” truck includes both types 6 and 7 from Figure 2-1, and a “class 8” truck is any tractor-trailer combination.

Type 3: Two-axle, four-tire, light trucks

Type 5: Two-axle, six-tire, medium trucks

Type 6: Three-axle single unit trucks

Type 7: Single unit trucks with > 3 axles

Type 8: Four or fewer-axle single-trailer trucks

Type 9: Five-axle single-trailer trucks

Figure 2-1. Truck classes defined by FHWA.

A vehicle classification scheme used in several states is based on dividing vehicles into four or five length classes. For example, a common four-class length structure is: < 26 ft., 26-39 ft., 39-65 ft. and > 65 ft. Vehicles less than 26 ft. long are assumed to be automobiles (although this grouping also would capture motorcycles and light trucks from class 3 in the FHWA scheme). The 26-39 ft. class includes single-unit trucks (FHWA types 5-7). The 39-65 ft. class includes most tractor-trailer trucks (FHWA classes 8 and 9). Vehicles over 65 ft. long are either oversize loads or trucks with multiple trailers (FHWA classes 10-13).

2.2 Sensors and Traffic Data

Estimation of O-D matrices was originally defined as a problem in which the only data available were traffic counts on links. The basis for this assumption was that counts from loop detectors in the pavement are the most commonly collected type of traffic data. Over the years of development of O-D estimation models, some efforts have made allowance for a few other types of data, including cordon counts, intersection turning movements, data collected at toll booths, and automatic vehicle identification (AVI) data, but in general there has been very limited interest in using these additional data types.

Because the current modeling effort is designed to use a wide variety of data types in the O-D estimation process, some likely types of data, and the sensors used to collect them, are discussed.

2.2.1 Loop Detectors and Link Count Data

A link count is simply the number of vehicles crossing a specific link over a defined time interval. This type of data is ubiquitous because loop detectors buried in the pavement are widely deployed as the basic data collection mechanism for traffic engineering. Figure 2-2 illustrates the basic set-up and technology of a simple loop detector. The loop is a few turns of wire embedded in the pavement, which forms an inductor. When a vehicle passes over the loop, the mass of metal in the vehicle changes (reduces) the inductance, and a circuit senses this as a pulse with a specific duration. If the width of the detector is d, and a vehicle (indexed by i) of length L_i traveling at a speed u_i passes over the detector, the duration of the pulse is $\frac{L_i + d}{u_i}$. Since we can measure the duration of the pulse, if we knew either the speed or the vehicle length, we can determine the other. However, we cannot determine both speed and vehicle length simultaneously from the detector output.

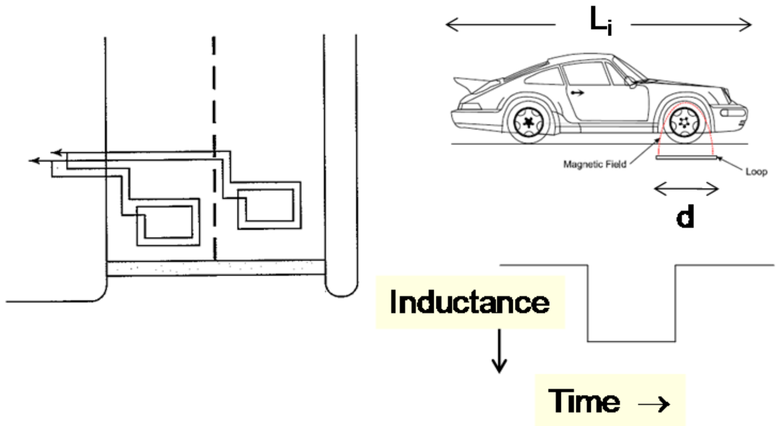

Figure 2-2. Simple single-loop detector implementation.

The usual output from a loop detector is a count and occupancy over some defined time period, *T* (often 20 sec or 30 sec). The count is simply the number of pulses recorded, or the number of vehicles counted. Occupancy is the proportion of time that there is a vehicle present over the detector. The inductance recorded at the detector over time is illustrated in Figure 2-3. If there are *n* pulses recorded over time *T*, then the occupancy measurement is:

$$Occ = \frac{\sum_{i=1}^{n} \frac{L_i + d}{u_i}}{T} \quad . \tag{2-1}$$

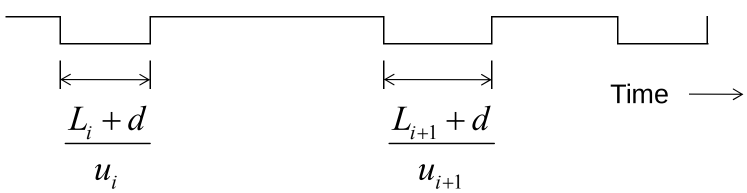

Figure 2-3. Loop detector recording over time.

If an average vehicle length, *L*, is assumed, then occupancy is directly related to the traffic density (vehicles/mile), and average speed is tied to density and counted volume, so the usual use of the detector measurements is to report traffic volume, *q*:

$$q = \frac{n}{T} \tag{2-2}$$

and the average speed of that traffic, *u*:

$$u = \frac{q\left(L+d\right)}{Occ} \quad . \tag{2-3}$$

However, since an important aspect of vehicle classification is sensing vehicle length, the *a priori* assumption of the average vehicle length in the traffic stream makes the use of single-loop detectors for vehicle classification highly problematic. A few authors have tried to create ways around this limitation. Sun and Ritchie (1999) and Jeng and Ritchie (2008) had some success using more detailed waveform output from the detectors. However, most detectors deployed in the field are not capable of producing that waveform output, so implementation of their ideas would require some extensive retro-fitting. Wang and Nihan (2003) created a two-bin classification model ("cars" and "larger") by exploiting an ability to estimate average speed separately from the loop detector output. Eq. 2-3 can be rearranged to solve for average vehicle length if average speed is known:

$$L = \frac{u\left(Occ\right)}{q} - d \quad . \tag{2-4}$$

If there are only two length classes, with known average lengths L_1 and L_2, and a proportion λ of the vehicles are in class 1, then the overall average length must be $L = \lambda L_1 + (1-\lambda)L_2$. If we combine this result with eq. 2-4, we can use the single-loop detector output to estimate λ. This, in turn, allows the total volume q to be broken down into two class counts. However, this only works when there are two classes, and if we assume that the average lengths in each class are known.

Zhang, *et al*. (2006) created a series of neural network models to attempt classification of vehicles into length groupings based on a series of single-loop detector measurements. They define a neural network for each of four vehicle length classes (< 26 ft., 26-39 ft., 39-65 ft. and > 65 ft.). Each network takes as input a set of 18 measurements (volume and lane occupancy over nine previous 20-sec. intervals, or three minutes worth of data), plus a time stamp. Each network produces an estimate of how many vehicles in a particular length class passed over the detector during that 3-minute span. They found that they could estimate the volume in the first bin (< 26 ft.; i.e., passenger cars and light trucks) reasonably accurately, but that the estimated volumes in the three bins of larger vehicles (medium and heavy trucks) had quite substantial errors. Thus, their approach seems potentially capable of separating cars from larger vehicles (i.e., two classification categories), but does not seem to be an effective way of obtaining any more detailed distinctions of truck volumes in different size classes.

Dual-loop detectors are a much more reliable means of obtaining length-based vehicle classification counts, although the deployment of dual-loop detectors is more limited. A dual-loop detector setup across a three-lane freeway section is illustrated in Figure 2-4. As a vehicle passes over a dual-loop detector, it creates two pulses, one from each loop. The beginning and end of each pulse can be labeled as a time (t_1 through t_4), as illustrated in Figure 2-5.

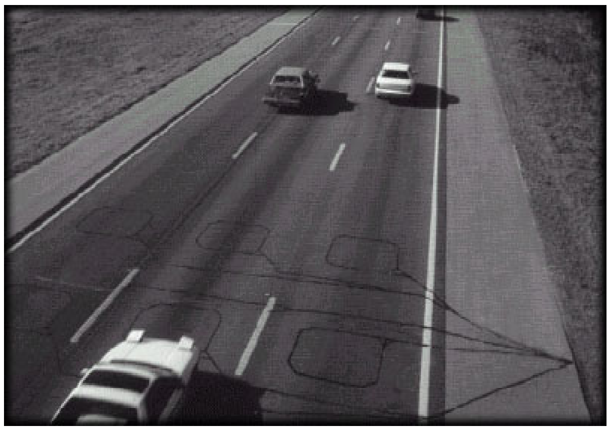

Figure 2-4. Dual-loop detector installation.

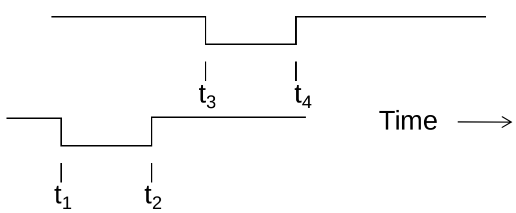

Figure 2.5. Dual-loop detector inductance pulses as a vehicle passes.

Then if the known distance between the two loops is D, the speed of vehicle i passing over the detector is:

$$u_i = \frac{D}{t_3 - t_1}. \tag{2-5}$$

Given this speed, the vehicle length can be estimated based on the average duration of the two pulses:

$$L_i = u_i \left[\frac{(t_2 - t_1) + (t_4 - t_3)}{2} \right] - d \ . \tag{2-6}$$

Vehicles of different lengths can then be grouped into some set of classes and counted. The length-based classification scheme (four vehicle length classes: < 26 ft., 26-39 ft., 39-65 ft. and > 65 ft.) described earlier may be acceptable for rural freeways, but in urban areas it is likely that few multi-trailer combinations will be observed, and it would be useful to separate FHWA type 5 trucks (two-axle, six-tire) from larger single-unit trucks of FHWA types 6 and 7. However, it is difficult to separate those single-unit classes using length alone.

In relatively uncongested conditions, dual-loop detectors can provide useful data on vehicle length, but the error rate on classifications is often quite high. Nihan, *et al*. (2002) reported on experiments that showed approximately 30-40% of trucks are misclassified, using the four-bin scheme. The most common errors are placing bin 2 trucks in bin 3, and placing bin 3 trucks in bin 4.

In congested conditions, when vehicles may often stop over a detector, the quality of the data degrades considerably. Wei (2011) has recently developed some methods for length-based vehicle classification using loop detector data under stop-and-go traffic conditions, but this remains a troublesome issue, of particular concern with loop detectors on urban streets.

2.2.2 Cordon Counts

Cordon counts identify some geographic area of interest and count all vehicles crossing the "cordon" enclosing that area (entering, leaving or both). These counts are usually done for specific purposes, and are often performed manually over a relatively short period. They are not normal ongoing data collection methods. However, when such counts are available, they may offer useful observations of total origins or total destinations within some defined area. This area may or may not correspond to one or more zones defined in the O-D table, however.

2.2.3 Turning Movement Data

Turning movements at intersections relate inbound and outbound flows on different legs (or approaches) to an intersection. Using the simple intersection in Figure 2-6 as an example, turning movements for all vehicles entering on approach 1 (I_1), would specify the proportions of those vehicles that turned right, went straight through, and turned left.

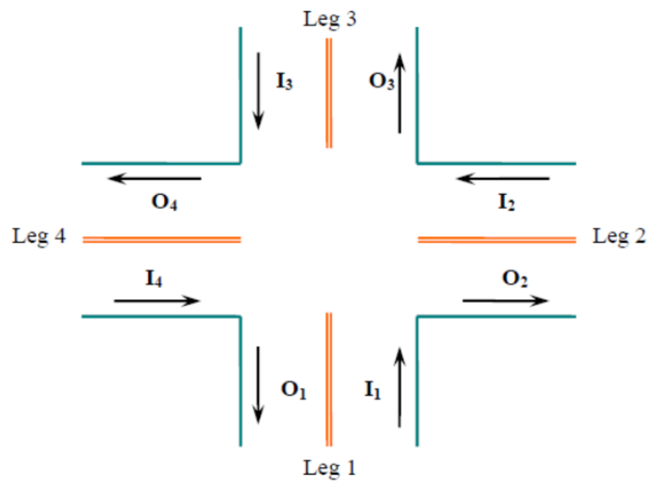

Figure 2-6. Simple four-leg intersection with inbound and outbound counts.

If data are collected on all movements at a given intersection with *N* approaches, the inbound and outbound counts can be related as shown in eq.2-7 (assuming no U-turns) :

$$\begin{bmatrix} O_1 \\ O_2 \\ .. \\ O_N \end{bmatrix} = \begin{bmatrix} 0 & \alpha_{21} & .. & \alpha_{N1} \\ \alpha_{12} & 0 & .. & \alpha_{N2} \\ .. & .. & .. & .. \\ \alpha_{1N} & \alpha_{2N} & .. & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ .. \\ I_N \end{bmatrix} \tag{2-7}$$

where α_{ij} is the proportion of traffic entering on leg *i* that exits on leg *j*.

Turning movements are not usually collected on an on-going basis as normal traffic data because in most cases, collecting the data requires manual observers (either to record data in the field or to watch video offline). However, these counts are important for optimal signal timing and thus may be collected periodically at a given intersection as part of a project to improve signal timing and intersection operation. For normal traffic engineering use, the total counts of vehicles making various turns are the important data, and it is not common to collect counts for vehicles in different classifications separately.

The potential value of turning movements for O-D estimation is that they specify a sequence of links used by a vehicle, rather than just counts on individual links. This provides an element of path information. In most previous O-D estimation efforts, the underlying assumption governing vehicle flows on the network is deterministic user equilibrium, and path flows are not uniquely identified by the equilibrium conditions, so this type of partial path information has not been of great interest. However, if the underlying flow pattern is assumed to be a stochastic user equilibrium, there is a set of identifiable path flows associated with the equilibrium, and this connection makes turning movements of significant potential value for the O-D estimation process.

A multiclass O-D estimation process also creates interest in having the turning movements separated by vehicle class. If the turning movement data are collected manually, this is certainly possible even though not commonly done. However, because manual data collection is expensive and not done frequently at any given intersection, there is also interest in automated methods of collecting turning movement data with vehicle classification that could be in place on a continuing basis.

Video data collection and processing is capable of making turning movement counts. Miovision (www.miovision.com), for example, markets a portable video data collection system designed for short term deployment at an intersection, as a substitute for manual data collection. The video processing software associated with this system is asserted to be able to provide vehicle classification data (at least at the level of separating medium trucks and heavy trucks from buses and automobile traffic, although the in-field assessments of count accuracy (see, for example, Swann (2010) and Schneider (2011)) have assessed only total counts. The accuracy of this system for total vehicle counts is quite high (generally with no more than 1-2% error), but no validation checks on classification counts have been identified in our analysis.

Further development of accurate and effective video collection and processing for turning movement data by vehicle class appears likely to be a useful endeavor, particularly when combined with a multiclass O-D estimation capability.

2.2.4 Toll Booth and Other Forms of Partial Path Data

Partial path data is sometimes available from special surveys at specific facilities. For example, truck drivers in line at a toll booth may be questioned for “last stop” and “next stop” information. This is done occasionally at locations like the toll bridges crossing the Hudson River into New York City, but collection of data in this way is labor intensive, expensive and prone to substantial reporting errors. Another example of such data is collected for U.S. import statistics. Counts of containers entering a specific port and destined for somewhere in the U.S. are collected by origin country. However, the specific destination inside the U.S. is not reported.

In urban networks this type of data can also be collected by video, using license plate matching software. That is, a specific truck may be observed at several points within a network at different times. By matching the data collected by different cameras, partial path information for the truck can be inferred. We may not know the original origin (or the final destination) of the truck, but we can infer some information about their path through the network, and whether stops within the network (which define destinations and origins of "linked" trips) are likely to have been made.

2.2.5 Automatic Vehicle Identification (AVI) Data

An increasing number of trucks have GPS receivers and many are capable of reporting their location to some central dispatching center on a regular basis. This type of data is not normally available outside the company that operates the trucks, but it is potentially a very rich data source. There has also been interest in using this type of data from automobiles (an increasing proportion of which are equipped with GPS capability) to estimate O-D information for passenger vehicles.

2.2.6 Non-Traffic Data and Use of *a priori* Trip Tables

One of the strategies used in O-D estimation since the 1970's has been to use a "target" trip table within the process. This stems from the fact that link volumes by themselves are generally insufficient to uniquely identify the trip table from which they arose (i.e., the inverse problem does not have a unique solution). One way to deal with this non-uniqueness is to postulate a trip table and then use the link volumes to find the trip table that is "closest" to this *a priori* table, and still consistent with the observed link data. Thus, the observed link data is used to adjust this *a priori* solution, rather than to dictate a solution on its own.

However, this approach depends on having some *a priori* O-D estimate available to use as a "target." In some cases, there may be an old O-D estimate that is to be updated with new data, and the target trip table approach can work quite well. Because urban areas in the U.S. have been charged with making metropolitan transportation plans for some time, there is often an automobile trip table available from a previous iteration of the planning process. However, this is more troublesome for trucks because most transportation plans have not been too concerned with truck flows in the past.

For truck flows, another approach that may be useful is to use socio-economic and land use data to estimate truck trip ends (total origins and total destinations, by zone). For example, the Southern California Association of Governments (SCAG) has developed estimates of trip rates related to employment in seven different industries as well as to numbers of households in an area (described in U.S. Federal Highway Administration, 2007). Although their estimates were created for use in the Los Angeles / Long Beach area, such estimates may also be useful in other areas. These rates, which separate light, medium and heavy trucks, are shown in Table 2-2.

Table 2-2. Truck trip rates estimated based on employment and numbers of households in traffic zones.

	Outbound				Inbound			
	Lt	Med	Hvy	Subtotal	Lt	Med	Hvy	Subtotal
Households	0.0390	0.0087	0.0023	0.0500	0.0390	0.0087	0.0023	0.0500
AMC	0.0513	0.0836	0.0569	0.1919	0.0513	0.0836	0.0569	0.1919
Retail	0.0605	0.0962	0.0359	0.1925	0.0605	0.0962	0.0359	0.1925
Government	0.0080	0.0022	0.0430	0.0533	0.0080	0.0022	0.0430	0.0533
Manufacturing	0.0353	0.0575	0.0391	0.1319	0.0353	0.0575	0.0391	0.1319
Transportation	0.2043	0.0457	0.1578	0.4078	0.2043	0.0457	0.1578	0.4078
Wholesale	0.0393	0.0650	0.0633	0.1677	0.0393	0.0650	0.0633	0.1677
Service	0.0091	0.0141	0.0030	0.0262	0.0091	0.0141	0.0030	0.0262

Notes:

Lt = light trucks (8,501-14,000 lb GVW) = FHWA class 3

Med = medium trucks (14,001-33,000 lb GVW) = FHWA class 5

Hvy= heavy trucks (>33,000 GVW) = FWWA classes 6 and higher

AMC = Agriculture/Mining/Construction

*Household trip rates are daily trips per household; all other trip rates are daily trips per employee

To use these trip rates, it is necessary to estimate employment and numbers of households within defined origin-destination zones. Then the trip rates can be used to construct "row totals" and "column totals" for the O-D matrices of truck classes.

2.3 Conclusions

Traffic data is available from a variety of sensor types, and the range of possibilities is increasing as better (and less expensive) video processing becomes available, and as GPS-based vehicle location technology becomes more widely deployed. Most of the O-D estimation methods that have been developed since the 1970's are focused on using link counts because that data (often obtained via loop counters in the pavement) is most widely available. Most deployed loop counters are single-loop installations that cannot reliably distinguish vehicle classes, so the O-D estimation methods have primarily been single-class models.

In this project, the primary objective is to explore formulation and solution methods for a multi-class O-D estimation process. This offers the opportunity to better understand the flow patterns of different truck size classes, as well as the differences between truck O-D patterns and automobile O-D patterns. To support such a process, better vehicle classification data is vital, and this chapter has discussed some of the possible technologies for obtaining such data. The following chapter describes a model formulation for O-D estimation that can use various types of data.

Chapter 3

MODEL FORMULATION AND SOLUTION

3.1 Model Formulation

The problem of determining an O-D matrix from observed traffic flow data is generally underdetermined because there are typically many more unknowns (O-D matrix entries) than there are observations. This means that there are many different O-D tables that could produce the same set of observed link volumes. As a result, one aspect of O-D matrix estimation is defining what is to be considered the “best” O-D matrix from among the candidates. A common approach to this issue has been to assume some “target” matrix (often an old O-D matrix to be updated), and defining the best updated matrix as the one that is closest to the old matrix, but also consistent with the updated link volumes. However, for the multi-class model considered here, reliance on the existence of some previous O-D matrix (or set of matrices for the various vehicle classes) is problematic. This is particularly true for trucks, for which existing data is often quite sparse and no previous O-D tables may have been estimated separately. Thus, the model developed here does not assume existence of any previous O-D table, although they can be inserted if available.

A second important aspect of the problem is that observed data, whether traffic counts on links, classification counts from video data, turning movements, etc., always contain errors. As a result, the data may be internally inconsistent and the model for estimating an O-D matrix must be robust in the face of “noisy” or inconsistent data. An effective way of incorporating that robustness is to minimize the squared errors or deviations from the observed values, but to allow estimates that do not match observed values exactly.

A third critical aspect of the O-D estimation problem is the assumption of how a link flow pattern on the network is constructed from the O-D flows. The collection of trip rates is a matrix, Q, with elements q_{ij} representing movement from origin zone i to destination zone j. The arc (or link) flows are denoted x, with the flow on a specific arc, a, denoted as x_a. One possible assumption is that there is a set of constant, volume-independent, path choice proportions for each O-D pair that can be determined *a priori* and used in the inverse model, creating a linear relationship between Q and x. This is called a proportional assignment assumption. It is useful in uncongested networks, and was widely used in the O-D estimation methods developed in the 1980’s.

A second common assumption is that link flows in the network are determined via a deterministic user equilibrium process. The equilibrium concept is important in congested networks. However, it makes the O-D estimation process much more difficult because an important parameter of the process, the route-choice proportion, depends on the O-D matrix being estimated. Early efforts to incorporate equilibrium effects were discussed by Turnquist and Gur (1979) and Nguyen (1984). Deterministic user equilibrium (DUE) is a widely used mechanism for predicting flows in congested networks, and this makes it an attractive option for O-D estimation as well. Initially, it was difficult to accommodate both the concept of DUE (which assumes perfect knowledge of travel impedances by drivers) and the fact that link data are invariably at least somewhat inconsistent with that assumption, but Sherali, *et al*. (1994) constructed an elegant approach to deal with that problem when there is a single vehicle class present.

A third possibility for the mechanism of determining link flows from O-D volumes is to assume a stochastic user equilibrium (SUE). The SUE assumption allows for errors in perception on the part of drivers as they make route choice decisions, and thus offers a richer view of equilibrium flows in the network. Daganzo (1982) showed that the equilibrium conditions for an SUE solution can be written as the following set of nonlinear equations:

$$x_a^m - \sum_{ij} q_{ij}^m p_{ij}^{am}\left(Q,c\right) = 0, \qquad a = 1,\ldots,n; \quad m = 1,\ldots,M \tag{3-1}$$

where x_a^m is the class m volume on link a, and $p_{ij}^{am}\left(Q,c\right)$ represents the link utilization coefficients (i.e., the fraction of the class m O-D volume for pair ij that appears on link a). These link utilization coefficients are functions of both the vector of costs on the links of the network (c), and of the O-D volumes themselves (Q), representing all vehicle classes.

If we have observed link volumes (on some, but not necessarily all, links), the conditions in equation (3-1) provide a way of connecting that data to the estimates of O-D volumes. However, doing this in a computationally practical way is non-trivial. Several authors have approached this issue (for a single vehicle class) using a bi-level programming formulation, where the upper-level problem adjusts the q_{ij} values, and the lower-level problem finds link flows, given the q_{ij}'s. Maher, *et al*. (2001) developed a bi-level programming approach for a logit-based SUE formulation and one vehicle class.

The focus of the current project is to develop multi-class O-D estimation models, and it is important to relate this work to some previous multi-class efforts, particularly those oriented to truck O-D estimation. List and Turnquist (1994, 1995) proposed a method for estimating multi-class O-D tables for truck movements using proportional assignment assumptions, and applied the technique in the New York City metropolitan area. The procedure made use of several different types of data, including several types of vehicle classification counts that had been made for special purposes by different transportation agencies in the New York area. An extended version of this technique (List, *et al*., 2002) has been instituted as a part of the "Best Practice Model" for on-going use in the New York City area.

Another example of truck flow estimation, using a combination of link count data and regional input-output economic data for Ontario, is described by Al-Battaineh and Kaysi (2007). One of the interesting ideas represented by their work is that truck movements are connected to economic activity, so regional economic data may be of help in estimating truck flows. This builds on the general concept that link counts of traffic (truck or otherwise) are not the only useful source of data for estimating O-D tables, and we should be creating methods that can integrate a wider variety of data types.

A major difficulty with these "truck only" O-D estimation methods is that they do not account for the interactions of trucks and automobiles in congested networks. A more complete approach is to estimate separate O-D tables for several vehicle classes (automobiles and various truck size classes) simultaneously, accounting for the different operating characteristics of the various vehicle classes and their interactions in creating equilibrium flows in the network. However, this clearly is a more complex estimation problem than the single-class models, and depends on having some of the observed data (although not necessarily all) separated by vehicle class.

There has been very limited work on estimating O-D tables for multiple vehicle classes, but a noteworthy effort of this type is described by Wong, *et al*. (2005). They developed a formulation and solution algorithm for a five-class problem (autos, taxis, buses, light trucks, and heavy trucks) for use in Hong Kong, using deterministic equilibrium (DUE) assumptions.

Ha, *et al*. (2007) developed a multi-class estimation model for use in Seoul, Korea. Their method is based on minimizing squared errors between estimated and observed classification counts on network links. They compared the multi-class O-D estimation method against a single-class model using the resulting trip length distribution as a criterion, and concluded that the multi-class model performed much better, but they provide few details of the actual implementation.

Raothanachonkun, *et al*. (2005) propose a multi-class estimation model focused on automobiles, light trucks and heavy trucks as vehicle classes. Their approach has the interesting characteristic of being aimed at estimating dynamic O-D matrices, rather than static ones, and uses an assumption of stochastic equilibrium as the underlying model of flow assignment in the network. Although they sketch out the outline of the model, they provide no indication of actual computational implementation.

In a subsequent paper (Raothanachonkun, *et al*., 2006), the same authors analyze multi-class static O-D estimation using a genetic algorithm for solution. They provide illustrative results on a very small network, but no indication of how the procedure scales to larger networks.

Previous efforts at using stochastic equilibrium (SUE) assumptions in the flow prediction aspect of the O-D estimation problem have all been based on a logit model of route choice through the network (which produces the $p_{ij}^{am}(Q,c)$ coefficients). The logit model is attractive for its computational simplicity, but it imposes some relatively unrealistic assumptions on the route choice probabilities, as a result of basic assumptions in the choice structure on which the model is built. One of its obvious weaknesses as a route choice model is the assumption that the characteristics of routes are independent of one another, even when those routes may overlap for a large fraction of their length. Cascetta, *et al*. (1996) proposed a method for relaxing the route independence assumption, but it creates quite a complicated O-D estimation process.

An alternative approach is to assume a probit model of route choice. The underlying assumption of the probit model (that uncertain route characteristics, or driver perceptions of those characteristics, are approximately Normal) is intuitively appealing, but the computations required for assigning traffic to a network using probit-based methods are significantly more complex than the computations for logit-based assignment. This has inhibited more widespread use of the probit model for SUE predictions. However, we have decided that its conceptual advantages outweigh its computational issues, and the model developed here is built using a probit-based SUE assumption.

The SUE conditions represented in eq. (3-1) can be incorporated into the O-D estimation process in two different ways. One way is to incorporate the conditions directly as constraints into the optimization for determining the O-D elements. Maher, *et al*. (2001) refer to this approach as "equilibrium programming" (following terminology defined by Garcia and Zangwill, 1981). A second way is to formulate a bi-level model, where the upper-level problem adjusts the q_{ij} values, and the lower-level problem finds link flows, given the q_{ij}'s. If a logit-based SUE model is used, the partial derivatives of the link utilization coefficients can be constructed in a reasonably straightforward way, and an equilibrium programming approach can be effective. However, this is much more difficult with a probit-based model and the bi-level programming approach seems likely to be more effective. The bi-level approach has been adopted here.

Under ideal conditions, the link count data will separate each of the defined vehicle classes and there will be counts available for all network links. In practice, however, this is unlikely to be the case. The observed values for the link volumes may correspond to aggregations of the vehicle classes defined, and there may not be observations for all links. We will index the available observations by l, and the set of all observed link counts as $\breve{X}$. The count for observation l will be denoted $\tilde{x}_l$, and include vehicles from a set of classes, M_l. The relevant link for observation l will be denoted a_l. Then the observed link count data and the observations' connection to the O-D tables for individual vehicle classes under the SUE conditions, can be written as:

$$\tilde{x}_l = \sum_{m \in M_l} \sum_{ij} p_{ij}^{a_l m}(Q,c) q_{ij}^m \qquad \forall l \tag{3-2}$$

The general form of constraints representing non-link count data (for an observation indexed by l) can be expressed as:

$$\sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^m = B_l \tag{3-3}$$

where B_l is an observed value and the u_{ij}^{ml} values are known coefficients. Constraint (3-3) states that some linear combination of elements in the estimated O-D tables (for some subset of vehicle classes denoted M_l and a subset of the O-D pairs ij in the set N_l) should sum to an observed value. This is a very general form, because the set N_l may have any combination of ij pairs in it, and the set M_l may contain one or more vehicle classes. If a prior estimate of the O-D matrix for one or more vehicle classes is available, that information can be used via eq. (3-3), but the expression in eq. (3-3) is much more general.

The observed values, regardless of type, must be assumed to contain errors. Thus, there is likely to be no solution that satisfies all the constraints specified by eqs. (3-2) and (3-3) exactly. We allow violations of the constraints (3-2) and (3-3), but with penalties in the form of a squared-error term for each constraint. This leads to a formulation of the problem as follows:

$$\underset{q_{ij}^m \geq 0}{Min}\ f = \sum_{l \in \breve{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^m \right)^2 + \sum_{l \in \breve{X}} \tau_l \left(\tilde{x}_l - \sum_{m \in M_l} \sum_{ij} p_{ij}^{a_l m}(Q,c) q_{ij}^m \right)^2 \tag{3-4}$$

The sets $\breve{X}$ and $\breve{B}$ define the observed values ($\tilde{x}_l$ and B_l) for the link counts and non-link data, respectively. The incorporation of the weighting constants η_l and τ_l allows us to control the degree to which emphasis is placed on various observations within the observed data.

One of the primary difficulties in solving the optimization problem in equation (3-4) is that the p_{ij}^{am} values are determined as part of the assignment of the O-D trips, Q, for all the vehicle classes to the network to achieve a stochastic equilibrium flow pattern. That SUE assignment problem is itself an optimization, and that problem needs to be solved in order to have the parameters to evaluate the function in (3-4). This illustrates the bi-level character of the overall problem. Evaluation of the objective function for the “upper-level” problem (3-4) requires solution of the “lower-level” optimization (the SUE flow problem). The following section describes a method for accomplishing the solution of the minimization expressed in eq. (3-5).

3.2 A Solution Method

The general flow of information in the solution approach is that a trial solution for the upper-level problem (a set of q_{ij}^{m} values) is passed to the lower-level problem. Assignment of these trips via an SUE algorithm results in link and path flows, and a set of p_{ij}^{am} values that are consistent with the input q_{ij}^{m} values. These p_{ij}^{am} values are passed back to the upper-level problem and treated as constants while the q_{ij}^{m} values are adjusted to create a new trial solution to the upper-level problem. If the upper-level problem has converged (i.e., the q_{ij}^{m} values are not changing), a solution to eq. (3-4) has been reached. However, if the new trial q_{ij}^{m} values are different from the previous values, they are passed back to the lower-level problem, where new p_{ij}^{am} values are created.

For solution of the lower-level problem, a probit-based SUE algorithm, based on the work of Maher and Hughes (1997) and Connors, *et al*. (2007), is used. The SUE calculations are iterative, and depend on a core method for doing stochastic network loading (SNL). This can be accomplished using the Stochastic Assignment Model (SAM) algorithm first described by Maher (1992).

In a more formal sense, the algorithm for solution of eq. (3-4) is as follows:

1) Set $n \leftarrow 0$ (an iteration counter). Based on free-flow conditions for travel times and costs, use SAM to do a SNL for each vehicle class. The result of this loading is a set of p_{ij}^{am} values. Denote the collection of values as P_0. For some types of data, the associated u_{ij}^{ml} values also depend on P_0. Denote the entire collection of u_{ij}^{ml} values as U_0.
2) Using the values of p_{ij}^{am} from P_0 and u_{ij}^{ml} from U_0 (as constants), solve the problem:

$$\underset{q_{ij}^m \geq 0}{Min} \quad \sum_{l \in \breve{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^m \right)^2 + \sum_{l \in \breve{X}} \tau_l \left(\tilde{x}_l - \sum_{m \in M_l} \sum_{ij} p_{ij}^{a_l m} q_{ij}^m \right)^2$$

Denote this solution as Q_0.

3) Using the O-D tables Q_n for the various vehicle classes, do the SUE calculations to get a feasible solution for the network link volumes, denoted X_n, the link utilization values, P_n, and the associated data coefficients U_n. The collection (Q_n, X_n, P_n, U_n) will be termed a *consistent* solution. (Note that when $n = 0$, the P_n and U_n values computed in this step replace the initial values estimated in step 1.)

4) Using the values of p_{ij}^{am} from P_n and u_{ij}^{ml} from U_n (as constants), solve the problem:

$$\underset{y_{ij}^m \geq 0}{Min} \quad \sum_{l \in \breve{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} y_{ij}^m \right)^2 + \sum_{l \in \breve{X}} \tau_l \left(\tilde{x}_l - \sum_{m \in M_l} \sum_{ij} p_{ij}^{a_l m} y_{ij}^m \right)^2$$

Denote this solution as Y_n.

5) Using the O-D tables Y_n for the various vehicle classes, do the SUE calculations to get a feasible solution for the network link volumes, denoted V_n. This will be termed a *trial* solution.

6) Linearize the assignment map between the two feasible solutions so that we can express potential trip tables as a function of a step size, □, where $0 \leq \gamma \leq 1$:

$$Q(\gamma) = Q_n + \gamma \left(Y_n - Q_n \right)$$

7) Perform a one-dimensional search on □ to find:

$$\underset{0 \leq \gamma \leq 1}{Min} \quad \sum_{l \in \breve{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^m(\gamma) \right)^2 + \sum_{l \in \breve{X}} \tau_l \left[\tilde{x}_l - p_{ij}^{a_l m} q_{ij}^m(\gamma) \right]^2$$

The values of u_{ij}^{ml} and $p_{ij}^{a_l m}$ are the values associated with the solution Q_n. Denote the optimal value of □ as γ^*.

8) If the difference $Q(\gamma^*) - Q_n$ is small enough, the solution has converged, and we can stop, using the last consistent solution as the final solution. If the process is not converged, make $n \leftarrow n + 1$, and $Q_n = Q(\gamma^*)$. Go to step 3.

Further discussion of the steps in this algorithm is facilitated by an example, so a small network is described in the following sub-section.

3.3 An Example Network

Figure 3-1 shows a small network with nine nodes and 12 two-way links. The values alongside the links are lengths, in miles. For analysis purposes, each of the two-way links is replaced by a pair of directional links, so the network has 24 directed links.

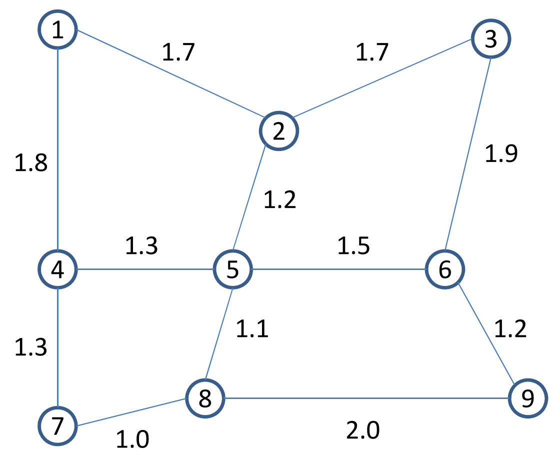

Figure 3-1. Example network.

Each link has a delay function to relate travel time to flow volume. Some commonly used delay function forms are the FHWA function, shown in eq. (3-5) and the Akçelik function (Akçelik , 1991) shown in eq. (3-6). Both functions are defined for all non-negative flows. Either form could be used in this example, but we will choose to assume the Akçelik functions. In this form of delay function, when flow exceeds capacity (i.e., $q_a > 1$), the delay increases rapidly, representing transient queuing on the link. For this example, we will assume $U_a = 1200$ veh/hour on all links, a free-flow speed of 40 mph, and $B = 0.4$. Figure 3-2 shows travel time for a link that is 1.0 miles in length, as a function of traffic flow on that link, using these parameters.

$$t_a(x_a) = t_{0a}\left[1+\alpha(q_a)^\beta\right] \tag{3-5}$$

$$t_a(x_a) = t_{0a} + \left\{0.25T\left[(q_a - 1) + \sqrt{(q_a-1)^2 + \frac{8B}{K_a T}q_a}\right]\right\} \tag{3-6}$$

where: t_{0a} = free-flow time for link *a* (e.g., length divided by the speed limit)

T = period under analysis (in this example, 1 hour)

x_a = flow on link *a* (e.g., veh/hour)

K_a = capacity of link *a* (in same units as flow variable)

q_a = x_a / U_a (saturation level)

B = parameter that varies by link type

α, β = parameters (often $\alpha = 0.15$ and $\beta = 4$).

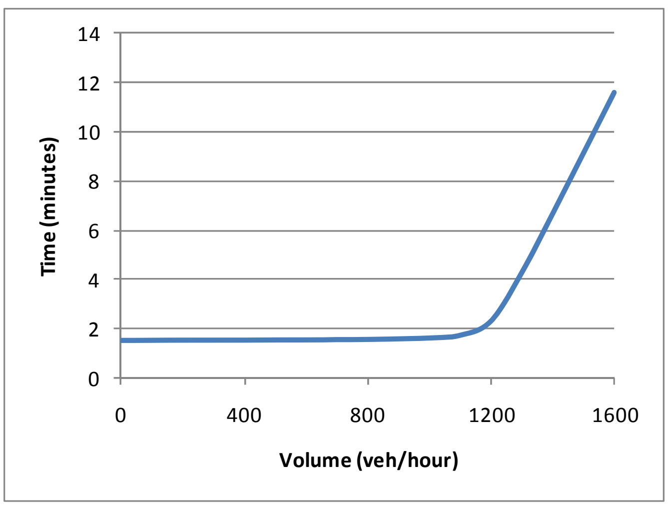

Figure 3-2. Travel time as a function of volume for a 1.0-mile link.

For this example, we will assume three vehicle classes – automobiles and two truck classes. The first truck class is two-axle, six-tire medium trucks (Type 5 shown in Figure 2-1). The second class includes all heavier trucks (Types 6-9 in Figure 2-1). Light trucks (Type 3 in Figure 2-1) are included with the automobiles. For the purposes of the multiclass network flow assignment, we define a *standardized flow*, z_a, as a weighted combination of the flows of the three classes, x_a^m :

$$z_a = x_a^1 + 1.8\, x_a^2 + 2.4\, x_a^3 \tag{3-7}$$

The standardized flow converts the truck flows into passenger car equivalents. Passenger cars (class 1) have a weight of 1. A medium truck (class 2) is assumed to be equivalent (in terms of contribution to volume-related congestion) to 1.8 passenger cars, and a heavy truck (class 3) is equivalent to 2.4 passenger cars.

The link delay functions for this example are written in terms of the standardized flow, z_a, where L_a is the length of the link in miles and the resulting travel time is expressed in minutes:

$$t_a\left(z_a\right) = 1.5\, L_a + \left\{ 15 \left[\left(\frac{z_a}{1200} - 1 \right) + \sqrt{ \left(\frac{z_a}{1200} - 1 \right)^2 + \frac{3.2}{1200} \left(\frac{z_a}{1200} \right) } \right] \right\} \tag{3-8}$$

We will assume that there are four O-D pairs in this network: 1-9, 3-7, 7-3 and 9-1. Thus, the core problem is to estimate three O-D tables (one for each vehicle class), each of which contains four elements.

As drivers decide on paths from origin to destination, they are assumed to be attempting to minimize a cost that depends on both distance and travel time. The cost functions for each link (for the three vehicle classes) are:

$$c_a^1(z_a) = 0.5 L_a + 0.33\, t_a(z_a) \tag{3-9}$$

$$c_a^2(z_a) = L_a + 0.33\, t_a(z_a) \tag{3-10}$$

$$c_a^3(z_a) = 1.5 L_a + 0.5\, t_a(z_a) \tag{3-11}$$

The terms in eqs. (3-9) through (3-11) that relate to the length of the link, L_a, do not depend on the flow volume, but as flow, z_a, increases on a link, $t_a(z_a)$ increases (using equation (3-8)), and thus the cost of using that link increases to all vehicle classes, but at rates that are class-dependent.

3.4 Stochastic Network Loading (Algorithm Step 1)

The basis of equilibrium in networks is that drivers of individual vehicles are choosing paths through the network to minimize their own costs. In deterministic equilibrium, each driver is assumed to have complete and accurate information about the costs of all links as the basis for a path choice. In stochastic equilibrium, the perceptions of travel time (or cost) are assumed to contain errors. These errors are modeled as random variables, ς_a^m, and the (random) perceived costs of using link a are then:

$$\tilde{c}_a^1(z_a) = 0.5L_a + 0.33t_a(z_a) + \varsigma_a^1 \tag{3-12}$$

$$\tilde{c}_a^2(z_a) = L_a + 0.33t_a(z_a) + \varsigma_a^2 \tag{3-13}$$

$$\tilde{c}_a^3(z_a) = 1.5L_a + 0.5t_a(z_a) + \varsigma_a^3 \tag{3-14}$$

Along the links of a path from origin to destination, the costs add, so we can also consider the cost along a path, p, as a random variable:

$$\tilde{g}_p^m = g_p^m + \psi_p^m \tag{3-15}$$

where $g_p^m = \sum_{a \in p} c_a^m(z_a)$ is the deterministic portion of the path cost and $\psi_p^m = \sum_{a \in p} \varsigma_a^m$ is the random portion. It is usual to assume that the link random variables have zero mean, so ψ_p^m also has zero mean, and the expected path cost is g_p^m.

If we consider two alternative paths, p_1 and p_2, for a given origin and destination, the probability of a driver choosing path p_1 is the probability that path p_1 is perceived as having lower cost:

$$\Pr\left[\tilde{g}_{p_1}^m \le \tilde{g}_{p_2}^m\right] = \Pr\left[g_{p_1}^m + \psi_{p_1}^m \le g_{p_2}^m + \psi_{p_2}^m\right] \tag{3-16}$$

Re-arranging eq. (3-16) to put the random variables all on one side of the inequality, we can write it as:

$$\Pr\left[\tilde{g}_{p_1}^m \le \tilde{g}_{p_2}^m\right] = \Pr\left[\psi_{p_1}^m - \psi_{p_2}^m \le g_{p_2}^m - g_{p_1}^m\right] \tag{3-17}$$

Equation (3-17) is written in the form of the cumulative distribution function (CDF) of the random variable $\psi_{p_1}^m - \psi_{p_2}^m$: i.e., the probability that the random variable takes on a value no greater than $g_{p_2}^m - g_{p_1}^m$. Thus, the path choice probability depends directly on the distributional assumption made for the error terms. There are two common choices: the Gumbel distribution, which leads to the logit model for the path choice probabilities, and the Normal distribution, which leads to expressing eq. (3-17) as a cumulative Normal distribution (the probit model).

Expanding eqs. (3-16) and (3-17) to the case where there are more than two alternative paths means that we are concerned with identifying the smallest of a set of random variables, and if the probit model (i.e., Normally distributed errors) is assumed, this means the smallest of a set of Normal random variables. The central result for effective computations is Clark's Approximation (Clark, 1961), which says that if Y_i and Y_j are Normally distributed random variables with means μ_i and μ_j , variances σ_i^2 and σ_j^2 and covariance v_{ij}, then the random variable T defined to be the minimum of Y_i and Y_j is itself approximately Normal with mean μ_T and variance σ_T^2, where:

$$\mu_T = \pi_i\mu_i + \pi_j\mu_j - a\phi(\alpha) \tag{3-18}$$

$$\sigma_T^2 = \left(\mu_i^2 + \sigma_i^2\right)\pi_i + \left(\mu_j^2 + \sigma_j^2\right)\pi_j - \left(\mu_i + \mu_j\right)a\phi(\alpha) - \mu_T^2 \tag{3-19}$$

In eqs. (3-18) and (3-19), the quantities a and α are computed from:

$$a^2 = \sigma_i^2 + \sigma_j^2 - 2v_{ij} \tag{3-20}$$

$$\alpha = \left(\mu_i - \mu_j\right)/\,a \tag{3-21}$$

$\phi(.)$ is the standard Normal density function. The probability that it is Y_i that is smaller is π_i, given by:

$$\pi_i = \Phi(-\alpha) \tag{3-22}$$

$\Phi(.)$ is the standard Normal CDF. As a result of eq. (3-22), we also have:

$$\pi_j = 1 - \pi_i = \Phi(\alpha) \tag{3-23}$$

Finally, if Y_i and Y_j were correlated with some third random variable Y_k, then the covariance of T with Y_k is:

$$v_{Tk} = \pi_i v_{ik} + \pi_j v_{jk} \tag{3-24}$$

Equations (3-18) through (3-23) create a straightforward way of obtaining the mean and variance of the minimum of two Normal random variables. The fact that the minimum is also approximately Normal, combined with equation (3-24), further allows this result to be applied iteratively across an arbitrary set of Normal random variables to obtain the minimum of the entire set of *n* random variables. This makes the result of immense value in doing probit-based stochastic network flow assignment.

This is the idea exploited by Maher (1992) in constructing a Stochastic Assignment Model (SAM). Given a set of random link costs, $\tilde{c}_a$, assumed Normally distributed, SAM operates from an origin node and moves through a network, performing two basic operations – *scanning* and *merging*. Outbound links from a given node are scanned to construct the distributions of cost to reach the ends of those links. Sets of inbound links to a given node are merged to construct the distribution of cost required to reach that node. Once a node has been merged, the links outbound from it are eligible for scanning. When all inbound links to a node have been scanned, that node is eligible to be merged. Maher (1992) developed a very efficient algorithm for performing the required computations, based on Clark's (1961) results.

In concept, SAM calculates a set of probabilities that can be used to load all trips originating at the origin node (and terminating at all destinations) in a single pass. However, when doing so, it is quite easy to encounter a situation called "lock-up" in the algorithm, where no node is eligible to be merged and the algorithm cannot complete. This is not a problem in acyclic networks, but most transportation applications operate in networks where cycles are possible. It is possible to avoid the potential for lock-up by limiting the operations (and flows) to a set of "efficient" links, but doing so in a careful way forces the SAM algorithm to operate on a single O-D pair at a time, making it much more computationally intensive. However, for the current purposes, the benefits of using the concept of link efficiency seem to outweigh the additional computational burden, so this has been incorporated.

The concept of efficient links for an origin-destination pair originated with Dial (1971), who used the idea in his logit-based SNL algorithm. For a given origin-destination pair, crossing an efficient link moves one further from the origin and closer to the destination. Links that are not on the shortest path from the origin to the destination can be efficient, as long as they are on paths that are generally moving in the right direction. However, the concept precludes cycling in the network because somewhere along a cyclic sub-path one must be moving "backwards" – i.e., getting closer to the origin and further from the destination. Thus, if flows are limited to paths made up of only efficient links there can be no cycles in the paths. For a single O-D pair, this allows the SAM algorithm to avoid the potential problem of lock-up.

The idea of defining and using only efficient links can be illustrated using our simple example network. We'll define the costs for medium trucks (vehicle class 2) at a set of initial conditions when z_a = 0 for all links. Equation (3-8) is used to compute travel times (minutes) for all links, $t_a(0)$. Then equation (3-10) is used to compute the costs for medium trucks: $c_a^2(0)$. These costs are shown along the links in Figure 3-3. We consider the O-D pair 1-9, and Figure 3-3 also includes a set of node labels, C_O, representing the cost of the minimum-cost path from the origin (node 1) to that node. The minimum-path *tree* is indicated with the heavier directed links.

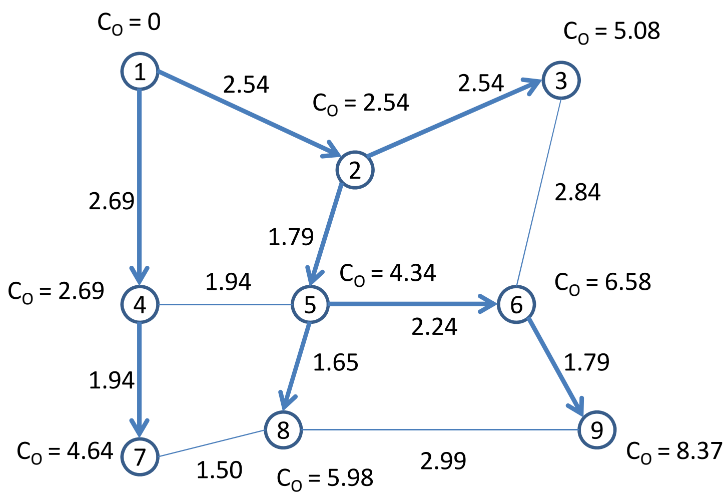

Figure 3-3. Example network with link costs and minimum path tree from node 1.

We can also construct the minimum-cost path from each node to the destination (node 9), and use the label C_D to indicate the cost from each node, as shown in Figure 3-4. The destination-focused minimum-path tree is also indicated in Figure 3-4, using the heavier directed arcs.

Figure 3-5 shows the combination of C_O and C_D labels at the nodes. The concept of an efficient link (for a specific O-D pair) is that traversing that link takes one further from the origin (C_O is increasing from the beginning node of the link to the ending node), and closer to the destination (C_D is decreasing from the beginning node of the link to the ending node). For example, we can see that traversing link 1→2 satisfies both criteria, and thus link 1→2 must be efficient for O-D pair 1-9. Figure 3-5 shows the collection of all efficient links for O-D pair 1-9.

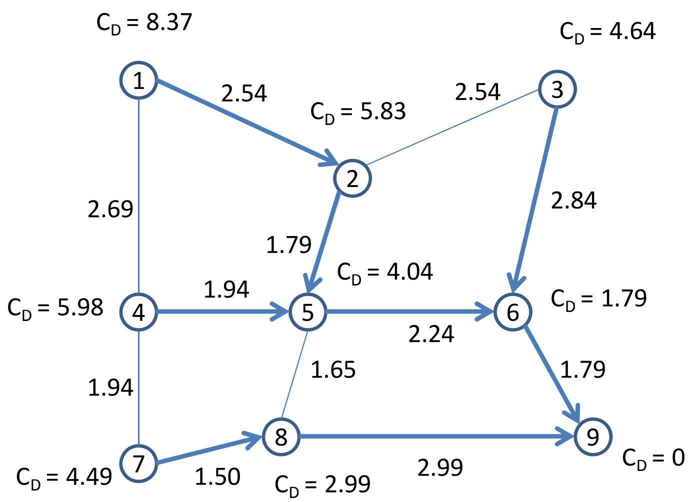

Figure 3-4. Example network with link costs and minimum path tree to node 9.

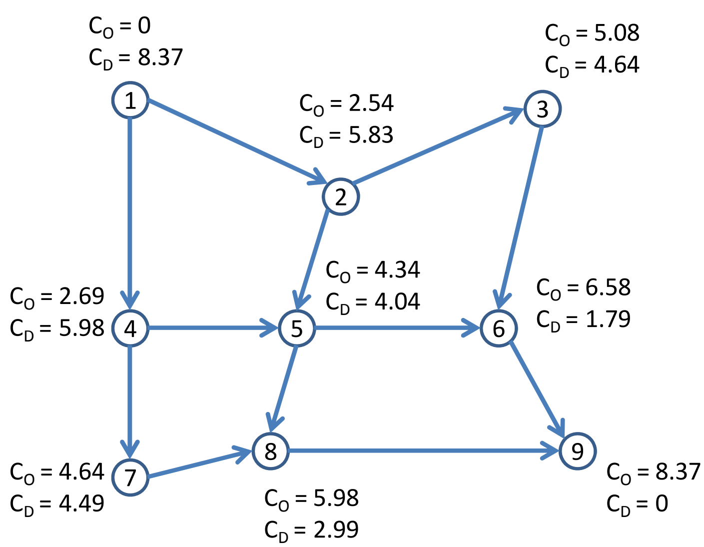

Figure 3-5. Example network with efficient links for O-D pair 1- 9.

Using only efficient links for the assignment of flow from 1-9 results in an acyclic network and avoids any potential problems of lock-up in the SAM algorithm. The set of efficient links also defines implicitly a set of possible paths between node 1 and node 9, but we have not had to explicitly enumerate those paths. The SAM algorithm, however, will produce probability values for use of links by each vehicle class on this O-D pair (the p_{ij}^{am} values noted earlier), as well as enough information to reconstruct probabilities of path use which will be important for constructing some constraints of the form described in equation (3-3). The results of the SAM calculation of link probabilities for O-D pair 1-9 and vehicle class 2 are shown in Figure 3-6.

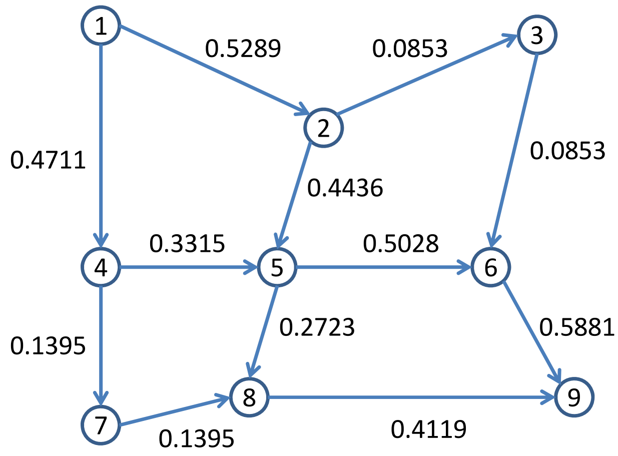

Figure 3-6. Link use probabilities for vehicle class 2 (medium trucks) and O-D pair 1- 9.

If we interpret these probabilities as proportions of a total flow of vehicles from origin to destination, we have conservation of flow at the nodes, and the product $p_{ij}^{am}\, q_{ij}^{m}$ represents part of the flow volume for link *a*. For example, if the O-D volume of class 2 trucks from 1 to 9 is 500, the expected volumes (rounded to integers) on the links of the network contributed by that vehicle class and O-D pair, are as shown in Figure 3-7.

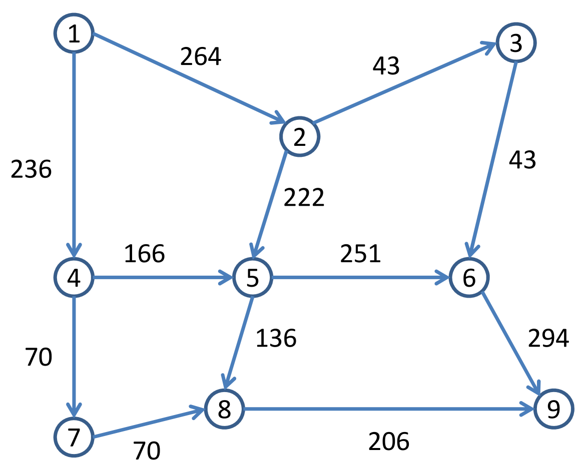

Figure 3-7. Expected flow volumes for vehicle class 2 (medium trucks) and O-D pair 1- 9, if the O-D flow is 500.

Of course, the total flow within each vehicle class on any network link may have contributions from many O-D pairs, and the observed counts on any link may include multiple vehicle classes, so in general, we have the relationship shown earlier as equation (3-2).

The link utilization probabilities also implicitly define probabilities for use of different paths between the origin and destination. Consider again the situation shown in Figure 3-6. If the probability of a truck using link 1-2 is 0.5289, that value must be the sum of probabilities for all paths that contain link 1-2. At node 2, that probability is split between paths that continue to node 3 and those that go to node 5. Thus, we can say that the probability of the partial path 1-2-3 is 0.0853, and the probability of the partial path 1-2-5 is 0.4436. At node 5, the partial path 1-2-5 merges with the partial path 1-4-5, and an assumption is necessary about how vehicles entering node 5 from different directions exit that node. This information may be provided by explicit turning probabilities (which could be calibrated from video surveillance of an intersection, for example). In the absence of such specific information, we might make a Markov assumption that each inbound partial path splits in the same proportions outbound. That is, since the ratio of the outbound probabilities at node 5 is $\frac{0.5028}{0.2723}$, each inbound partial path will divide in that proportion. Thus, the probability of the partial path 1-2-5-6 will be:

$0.4436\left(\frac{0.5028}{0.5028+0.2723}\right)=0.2878$, and the probability of the partial path 1-2-5-8 will be

$0.4436\left(\frac{0.2723}{0.5028+0.2723}\right)=0.1559$.

We continue splitting partial paths and computing probabilities until we reach the destination, at which point we have complete origin-destination paths identified. In this small example, there are six potential paths between node 1 and node 9, with usage probabilities computed as follows:

Path	**Probability**
1-2-3-6-9	0.0853
1-2-5-6-9	0.2878
1-2-5-8-9	0.1559
1-4-5-6-9	0.2150
1-4-5-8-9	0.1165
1-4-7-8-9	0.1395

It can easily be verified that these path probabilities sum to 1.0. We also note that the path (1-2-5-6-9) that was computed to be the shortest path (see Figure 3-3) has the highest probability of use. In this stochastic flow assignment, shorter paths have a higher probability of use, but less direct paths (like 1-2-3-6-9, for example) have non-zero probability of use. That is, not all traffic is assumed to use the single shortest path.

Being able to compute expected path probabilities for a given vehicle class and O-D pair allows us to take advantage of potential observations of vehicles at multiple locations. For example, suppose some number of class 2 trucks were observed on the sequence of links 2-5 and 5-8. Those trucks could be traveling from node 1 to node 9, using path 1-2-5-8-9, or they could be part of some other O-D pair (in this case, from 3 to 7, using path 3-2-5-8-7). The relative probabilities for the various paths give us useful information about how to factor such observations into our overall analysis.

3.5 Observations and Estimating O-D Tables (Algorithm Step 2)

To illustrate the use of count data to estimate an O-D table, let us suppose that traffic counts on links 1-4, 3-6, 5-6 and 5-8 are available. On link 5-6, the counts separate the vehicle classes in each direction. On links 1-4 and 5-8, the counts are from dual loop detectors and separate automobiles from trucks, but both classes of trucks are combined. Furthermore, on link 4-1, the truck count is missing. On link 3-6, counts are available in both directions, but are simply total vehicles (from a single loop counter). The 15 available observed data are as follows:

Link	Count
1-4	642 (automobiles – class 1)
1-4	48 (total trucks – classes 2 and 3)
4-1	654 (automobiles – class 1)
3-6	676 (total vehicles – all classes)
6-3	639 (total vehicles – all classes)
5-6	916 (automobiles – class 1)
5-6	34 (medium trucks – class 2)
5-6	45 (heavy trucks – class 3)
6-5	918 (automobiles – class 1)
6-5	43 (medium trucks – class 2)
6-5	67 (heavy trucks – class 3)

5-8	1004 (automobiles – class 1)
5-8	131 (total trucks – classes 2 and 3)
8-5	1006 (automobiles – class 1)
8-5	109 (total trucks – classes 2 and 3)

For this example, there are four O-D pairs for each vehicle class, or a total of 12 O-D entries to be estimated. Thus, we have more observations than unknowns, but the data may not all be consistent, so there may be no trip table that would match these observations exactly. Also, in large networks, there are normally many more O-D table elements than we have observations, so the problem is usually underspecified.

We use the SAM algorithm to compute link utilization probabilities and path probabilities for vehicle classes 1 and 3, just as for vehicle class 2 above, but the probabilities are different because the cost functions (3-9) and (3-11) for vehicle classes 1 and 3 are different from vehicle class 2. Figure 3-8 shows the link use probabilities for vehicle class 1 and O-D pair 1-9; Figure 3-9 shows the comparable values for vehicle class 3. These figures can be compared to Figure 3-6, which pertains to vehicle class 2. The probabilities for the other O-D pairs are also computed, but not shown explicitly in figures.

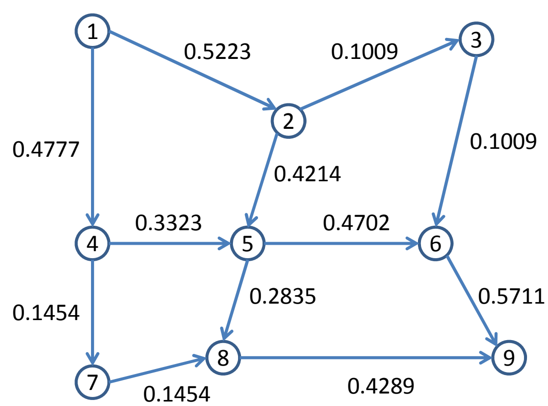

Figure 3-8. Link use probabilities for vehicle class 1 (automobiles) and O-D pair 1- 9.

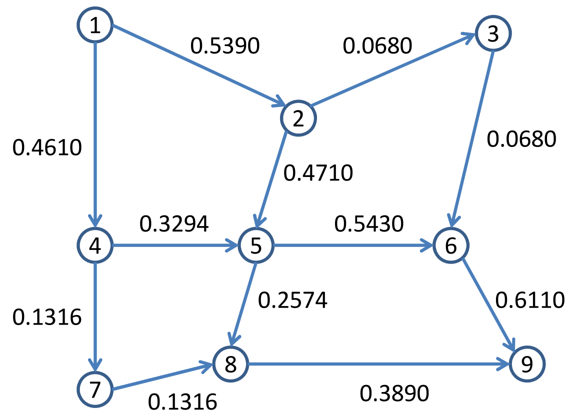

Figure 3-9. Link use probabilities for vehicle class 3 (heavy trucks) and O-D pair 1- 9.

The equations to relate the observed data to the unknown trip table values are the following:

$$642 = 0.4777\, q_{19}^{1} + 0.0577\, q_{37}^{1} + e_1$$

$$48 = 0.4711\, q_{19}^{2} + 0.0415\, q_{37}^{2} + 0.4610\, q_{19}^{3} + 0.0266\, q_{37}^{3} + e_2$$

$$654 = 0.0648\, q_{73}^{1} + 0.4805\, q_{91}^{1} + e_3$$

$$676 = 0.1009\, q_{19}^{1} + 0.4060\, q_{37}^{1} + 0.0853\, q_{19}^{2} + 0.3757\, q_{37}^{2} + 0.0680\, q_{19}^{3} + 0.3365\, q_{37}^{3} + e_4$$

$$639 = 0.3961\, q_{73}^{1} + 0.0964\, q_{91}^{1} + 0.3642\, q_{73}^{2} + 0.0809\, q_{91}^{2} + 0.3241\, q_{73}^{3} + 0.0640\, q_{91}^{3} + e_5$$

$$916 = 0.4702\, q_{19}^{1} + 0.2934\, q_{73}^{1} + e_6$$

$$34 = 0.5028\, q_{19}^{2} + 0.2786\, q_{73}^{2} + e_7$$

$$45 = 0.5429\, q_{19}^{3} + 0.2575\, q_{73}^{3} + e_8$$

$$918 = 0.2996\, q_{37}^{1} + 0.4656\, q_{91}^{1} + e_9$$

$$43 = 0.2848\, q_{37}^{2} + 0.4985\, q_{91}^{2} + e_{10}$$

$$67 = 0.2635\, q_{37}^{3} + 0.5387\, q_{91}^{3} + e_{11}$$

$$1004 = 0.2835\, q_{19}^{1} + 0.5530\, q_{37}^{1} + e_{12}$$

$$131 = 0.2723\, q_{19}^{2} + 0.6044\, q_{37}^{2} + 0.2574\, q_{19}^{3} + 0.6638\, q_{37}^{3} + e_{13}$$

$$1006 = 0.5518\, q_{73}^{1} + 0.2868\, q_{91}^{1} + e_{14}$$

$$109 = 0.6041\, q_{73}^{2} + 0.2748\, q_{91}^{2} + 0.6645\, q_{73}^{3} + 0.2589\, q_{91}^{3} + e_{15}$$

Using these 15 observations, we can estimate O-D tables for the three vehicle classes by solving:

$$\underset{q_{ij}^m \geq 0}{Min} \quad \sum_{l=1}^{15} \left(e_l\right)^2$$

The solution to the minimization problem (rounded to the nearest integer number of trips) is:

$$q_{19}^1 = 1199 \qquad q_{19}^2 = 50 \qquad q_{19}^3 = 41$$

$$q_{37}^1 = 1200 \qquad q_{37}^2 = 98 \qquad q_{37}^3 = 71$$

$$q_{73}^1 = 1200 \qquad q_{73}^2 = 32 \qquad q_{73}^3 = 88$$

$$q_{91}^1 = 1199 \qquad q_{91}^2 = 30 \qquad q_{91}^3 = 90$$

Because the link utilization probabilities from the SAM algorithm were constructed in step 1 based on free-flow conditions, and the trip table we have just estimated, if assigned to the network, will produce link volumes that are not free-flow, the entire process is not yet converged. That is the reason for steps 3-8 in the algorithm. However, we have constructed an initial estimate of the three trip tables that can be the basis for further iterations.

In this small example, the actual trip tables that were assigned to the network and used to generate the observed flows are known, even though in field applications of the O-D estimation process that is not the case. The actual O-D tables have not been used in any way in the estimation process, but we can compare the current estimated trip tables with the actual values to judge how similar they are. The actual trip tables are as follows:

$$q_{19}^1 = 1200 \qquad q_{19}^2 = 50 \qquad q_{19}^3 = 40$$

$$q_{37}^1 = 1200 \qquad q_{37}^2 = 100 \qquad q_{37}^3 = 70$$

$$q_{73}^1 = 1200 \qquad q_{73}^2 = 30 \qquad q_{73}^3 = 90$$

$$q_{91}^1 = 1200 \qquad q_{91}^2 = 30 \qquad q_{91}^3 = 90$$

It is clear that the initial estimates are very close to the true values, even though data were not available for all network links, some of the data aggregated vehicle classes, and the portion of the algorithm representing the SUE flow calculations has not yet been invoked.

Although in this example the trip tables estimated based on link counts only are very close to the true values, the 9-node example network is also useful to illustrate the incorporation of turning count data at intersections. For this illustration, turning movement data from three of the four approaches at node 5 is used and the assumption is that those turning movements are total counts (i.e., not specific by vehicle class). The available data are summarized in Figure 3-10. Turning movements are not specified for the approach from node 2. This helps illustrate that the computations can be done using any available data, and that not all approaches need to be covered.

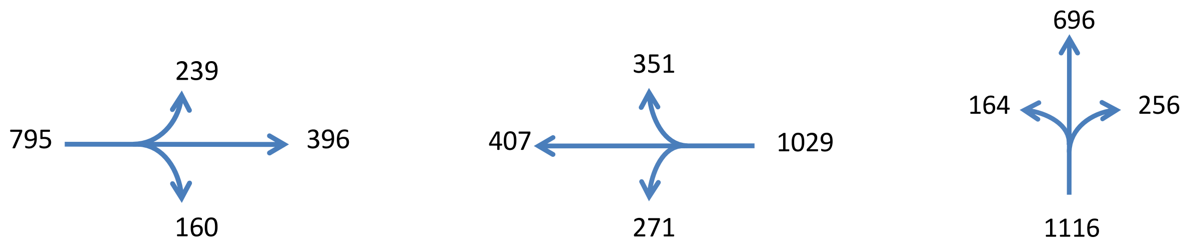

Figure 3-10. Available turning movement data at node 5.

A turning movement observation is equivalent to observing a vehicle on a sequence of two links. That is, the 396 vehicles that go straight through the intersection in the leftmost diagram in Figure 3-10 are vehicles that use the sequence of links 4-5 and then 5-6. This sequence is part of two paths for different O-D pairs:

O-D Pair	Path
1-9	1-4-5-6-9
7-3	7-4-5-6-3

As part of the SAM calculations, we have determined that the probabilities of these paths for the three vehicle classes on the respective O-D pairs are:

			Probabilities	
O-D Pair	Path	Vehicle Class 1	Vehicle Class 2	Vehicle Class 3
1-9	1-4-5-6-9	0.2073	0.2150	0.2234
7-3	7-4-5-6-3	0.0989	0.0845	0.0679

This allows construction of another observation constraint for the O-D table estimation:

$$396 = 0.2073\,q_{19}^{1} + 0.2150\,q_{19}^{2} + 0.2234\,q_{19}^{3} + 0.0989\,q_{73}^{1} + 0.0845\,q_{73}^{2} + 0.0679 q_{73}^{3} + e_{16}$$

The observations of turning movements are not simple link counts and the coefficients in the constraint are not link probabilities, so this is a constraint of the form in eq. (3-3) and the coefficients are examples of u_{ij}^{ml} values. Each of the movements illustrated in Figure 3-10 can be converted into an observational constraint of the general form given in equation (3-3):

$$239 = 0.1818\,q_{73}^{1} + 0.1785\,q_{73}^{2} + 0.1702\,q_{73}^{3} + e_{17}$$

$$160 = 0.1250\,q_{19}^{1} + 0.1165\,q_{19}^{2} + 0.1059\,q_{19}^{3} + e_{18}$$

$$351 = 0.2618\,q_{91}^{1} + 0.2867\,q_{91}^{2} + 0.3185\,q_{91}^{3} + e_{19}$$

$$407 = 0.1014\,q_{37}^{1} + 0.0864\,q_{37}^{2} + 0.0693\,q_{37}^{3} + 0.2038\,q_{91}^{1} + 0.2117\,q_{91}^{2} + 0.2202\,q_{91}^{3} + e_{20}$$

$$271 = 0.1982\,q_{37}^{1} + 0.1984\,q_{37}^{2} + 0.1943\,q_{37}^{3} + e_{21}$$

$$164 = 0.1255\,q_{91}^{1} + 0.1167\,q_{91}^{2} + 0.1058\,q_{91}^{3} + e_{22}$$

$$696 = 0.3579\,q_{73}^{1} + 0.4099\,q_{73}^{2} + 0.4750\,q_{73}^{3} + 0.1613\,q_{91}^{1} + 0.1581\,q_{91}^{2} + 0.1531\,q_{91}^{3} + e_{23}$$

$$256 = 0.1945\,q_{73}^{1} + 0.1941\,q_{73}^{2} + 0.1895\,q_{73}^{3} + e_{24}$$

When observations 16 -24 are added to the minimization problem, the resulting solution is:

$$
\begin{array}{lll}
q_{19}^{1} = 1199 & q_{19}^{2} = 48 & q_{19}^{3} = 42 \\
q_{37}^{1} = 1201 & q_{37}^{2} = 91 & q_{37}^{3} = 78 \\
q_{73}^{1} = 1200 & q_{73}^{2} = 34 & q_{73}^{3} = 85 \\
q_{91}^{1} = 1199 & q_{91}^{2} = 34 & q_{91}^{3} = 86
\end{array}
$$

This solution is very similar to the previous solution. In this example, the addition of turning movement data is not important for improving the solution, but the example does illustrate how non-link count data can be incorporated into the O-D estimation process. In subsequent experiments on a larger and more complicated network in Chapter 4, the value of turning movement data will be clearer.

Other types of partial path information may also be represented in similar constraints if the data acquisition equipment can provide it. For example, license plate matching capability could produce an observation that a certain number of vehicles were observed on both links 1-2 and 7-8 (i.e., nonsequential links). These two links will be parts of specific paths for which probabilities have been computed, and constraints of the same form as the ones above can be constructed. The key is in identifying the potential "efficient" paths within the SAM algorithm and then tying counts of vehicles observed on parts of those paths to the possible O-D pairs on which they could be traveling.

3.6 Consistent Solutions and Stochastic Equilibrium

If the O-D volumes resulting from the zero-flow estimates of link use probabilities were assigned to flow over the network, they would imply a set of link volumes that would create different link travel times and costs than those used to construct the link utilization probabilities, and thus the probabilities used so far are not consistent with the estimated O-D volumes. Step 3 in the overall algorithm described at the beginning of section 3 updates those probability estimates and resulting link volumes to be consistent with the current O-D table estimates, thus creating a collection of O-D volumes, link volumes and probabilities [denoted by (Q, X, P, U)] that are all internally consistent. This will be termed a *consistent solution* to the overall problem.

Finding a consistent solution depends on solving for stochastic user equilibrium (SUE) in the network -- a condition in which the link volumes, resulting travel times and costs, and estimated link utilization probabilities are all consistent with the behavioral principle that shorter (or faster or cheaper) paths for any given O-D pair should have higher probability of use than longer (or slower or more expensive) paths.

Finding the multiclass SUE solution is a process of solving the equilibrium conditions posed by Daganzo (1983) – eq. (3-1), repeated here for easy reference.

$$x_a^m - \sum_{ij} q_{ij}^m p_{ij}^{am} \left(Q, c\right) = 0, \qquad a = 1, \ldots, n; \quad m = 1, \ldots, M \tag{3-1}$$

Eqs. (3-1) may be viewed as the first-order conditions for an optimization problem, as described (for a single vehicle class) by Sheffi (1985). This optimization is:

$$\underset{x_a}{Min} = -\sum_a \int_0^{x_a} c_a(w)dw + \sum_a x_a c_a(x_a) - \sum_{ij} q_{ij} S_{ij}(c) \tag{3-25}$$

The first term in eq. (3-25) is the function that would be minimized at a deterministic user equilibrium solution. The second term is the total cost of travel in the network, and would be minimized at a system-optimal solution. The third term is the expected value of the total perceived network cost , using the Satisfaction function, $S_{ij}(c)$, defined as the expected perceived minimum cost of travel for O-D pair *ij*:

$$S_{ij}(c) = E\left[\min_k \tilde{g}_{ij}^k\right] \tag{3-26}$$

$\tilde{g}_{ij}^k$ is the perceived path cost for path *k* from origin *i* to destination *j*. Along the links of a path the costs add, so the cost along a path, *k*, is a random variable as described earlier. The expectation in eq. (3-26) is with respect to the distribution of the error terms in the perceived path costs, which depend on the link costs, so we write $S_{ij}(c)$ as a function of the set of link costs, *c*.

Maher and Hughes (1997) developed an algorithm for solving the optimization in eq. (3-25), taking advantage of the fact that the $S_{ij}(c)$ function is implicitly evaluated during the performance of the SAM algorithm. Their algorithm was originally designed for use with a single vehicle class, but the modification to include multiple vehicle classes is straightforward. Their algorithm involves iterative application of the SAM algorithm, combined with a search mechanism to average the successive solutions. Extending this process to include multiple vehicle classes doing the SAM calculations at each iteration for all vehicle classes separately, calculating a satisfaction function, $S_{ij}^m(c^m)$, function for each vehicle class, and updating link travel times for the next iteration based on the aggregate equivalent flow on each link.

Step 3 of the multiclass O-D estimation algorithm listed at the beginning of this chapter is the application of the multiclass version of the Maher-Hughes method for finding SUE solutions, using the current estimated trip tables, to produce the necessary consistent solution at a given iteration.

3.7 Solving the Nonlinear Optimization at Steps 2 and 4 of the Procedure

The construction of estimated trip tables, *Q*, with the link utilization probabilities, *P*, and coefficients, *U*, treated as constants, requires solution of a problem of the form:

$$\underset{q_{ij}^m \geq 0}{Min} \quad \sum_{l \in \bar{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^m \right)^2 + \sum_{l \in \bar{X}} \tau_l \left(\tilde{x}_l - \sum_{m \in M_l} \sum_{ij} p_{ij}^{a_l m} q_{ij}^m \right)^2 \tag{3-27}$$

This is a quadratic optimization, and for the solution to be sensible the trip table elements must be non-negative. One effective method for finding a solution to this problem is the L-BFGS-B algorithm (Byrd, *et al.* ,1995, 1997; and Morales and Nocedal, 2011). The core idea on which this algorithm is based is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method for nonlinear optimization. The L-BFGS version of this algorithm uses some approximations in the storage of derivatives to create a limited-memory (L) version of the method that is effective for large problems. The further extension to handle simple box constraints on variables (the B suffix) creates an effective method for the problem faced here.

The L-BFGS-B method is one of a class of algorithms called quasi-Newton methods. These algorithms are based on a core idea that if a function $f(\vec{q})$, where $\vec{q}$ is an *n* x 1 vector, is twice continuously differentiable, we can construct a quadratic approximation to it at a point $\vec{q}$ by using a Taylor series expansion around a point $\vec{q}^k$:

$$f(\vec{q}) \cong f(\vec{q}^k) + \left(\vec{q} - \vec{q}^k\right)\nabla f(\vec{q}^k) + \frac{1}{2}\left(\vec{q} - \vec{q}^k\right)^T \nabla^2 f(\vec{q}^k)\left(\vec{q} - \vec{q}^k\right) \qquad (3\text{-}28)$$

If $\nabla^2 f(\vec{q})$ is positive definite, then the minimum of the quadratic approximation occurs at the point $\vec{q}\,*$ where $\nabla f(\vec{q}*) = 0$. This point is found by solving:

$$\vec{q}* = \vec{q}^k - \left[\nabla^2 f(\vec{q}^k)\right]^{-1} \nabla f(\vec{q}^k) \qquad (3\text{-}29)$$

Newton's Method involves using eq. (3-29) iteratively, finding the next approximation $\vec{q}^{k+1}$ to the minimum of $f(\vec{q})$ by evaluating the gradient and the Hessian (matrix of second partial derivatives) of *f* at a point $\vec{q}^k$, and then computing the next estimate as:

$$\vec{q}^{k+1} = \vec{q}^k - \left[\nabla^2 f(\vec{q}^k)\right]^{-1} \nabla f(\vec{q}^k) \qquad (3\text{-}30)$$

The process is then repeated with $\vec{q}^{k+1}$ replacing $\vec{q}^k$.

In practice, Newton's method has several shortcomings. It requires storage and repeated computation of the inverse Hessian, and if the Hessian is ill-conditioned, the algorithm can be unstable or fail to converge. The techniques known as quasi-Newton methods have been developed to overcome these problems. The general ideas underlying quasi-Newton methods are described in several texts on nonlinear optimization and solution of nonlinear equations (e.g., Dennis and Schnabel, 1996; Luenberger and Ye, 2008). The central idea is to construct and update an approximation to the inverse Hessian in a way that does not require explicit inversion and that ensures that the approximation is always positive definite. Quasi-Newton method contains a whole family of algorithms. The difference lies in how they do the approximation of Hessian. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm has proven over time to be an effective and reliable means for doing that. The L-BFGS variation stores a simplified version of the whole *n* by *n* Hessian approximation matrix, using a history of the past *r* updates of the position and the corresponding gradient. L-BFGS-B, the bounded variation of L-BFGS, uses the idea of an active set to accommodate simple constraints. At every step, the algorithm identifies fixed and free variables using a simple gradient method and runs the L-BFGS only on the free variables.

3.8 The Search Process at Steps 6 and 7 of the Algorithm

At the end of step 5 of the algorithm, we have two feasible solutions to the O-D estimation problem – a current solution created through steps 2 and 3, and a new trial solution created in steps 4 and 5. The trial solution should be in a direction from the current solution that produces an improvement overall, but there is an important question of how far to move in that direction. This creates a search problem that operates along the line between the current solution and the trial solution.

To implement that search process, we linearize the assignment map between the two feasible solutions (the current solution Q_n and the trial solution Y_n) so that we can express potential trip tables and associated link volumes as a function of a step size, γ , where $0 \le \gamma \le 1$:

$$Q(\gamma) = Q_n + \gamma\left(Y_n - Q_n\right) \tag{3-31}$$

$$X(\gamma) = X_n + \gamma\left(V_n - X_n\right) \tag{3-32}$$

Then we can search for the value of γ that solves:

$$\underset{0\le\gamma\le1}{Min} \quad f(\gamma) = \sum_{l \in \breve{B}} \eta_l \left(B_l - \sum_{m \in M_l} \sum_{ij \in \mathrm{N}_l} u_{ij}^{ml} q_{ij}^{m}(\gamma) \right)^2 + \sum_{l \in \breve{X}} \tau_l \left[\tilde{x}_l - x_l(\gamma) \right]^2 \tag{3-33}$$

This is a one-dimensional optimization because the only variable is γ. We'll denote the optimal value of γ as γ^*.

Since $f(\gamma)$ is convex, we can solve for γ^* analytically by setting $f'(\gamma) = 0$. This gives us

$$\gamma^* = \frac{\sum_l \eta_l (B_l - c_l)(d_l - c_l) + \sum_l \tau_l (\tilde{x}_l - x_l)(v_l - x_l)}{\sum_l \eta_l (d_l - c_l)^2 + \sum_l \tau_l (v_l - x_l)^2} \tag{3-34}$$

where:

$$c_l = \sum_m \sum_{ij} u_{Qij}^{ml} \cdot Q_{ij}^m \tag{3-35}$$

$$d_l = \sum_m \sum_{ij} u_{Yij}^{ml} \cdot Y_{ij}^m \tag{3-36}$$

$$x_l = \sum_m \sum_{ij} p_{Qij}^{ml} \cdot Q_{ij}^m \tag{3-37}$$

$$v_l = \sum_m \sum_{ij} p_{Yij}^{ml} \cdot Y_{ij}^m \tag{3-38}$$

$$l \in \check{B}_l$$

$$m \in M_l$$

$$i, j \in N_l$$

These required calculations are straightforward and can be done quite easily.

3.9 Conclusions

This chapter describes a formulation of the multiclass O-D estimation problem that extends previous work in several important ways. It is constructed to accept a variety of different types of data that relate O-D volumes to observed values. This includes traditional link count data, but also accommodates other types of data such as turning movements at intersections, partial path observations from individual vehicles, etc. The data may be either class-specific or may aggregate a subset of vehicle classes. The very general structure of the way that observations enter the estimation process allows for a variety of possible data types, including partial path data from more advanced vehicle sensing, and socio-economic data related to trip origins and destinations. No prior O-D matrix is assumed, although one may be used if available (for any or all vehicle classes), through the construction in constraints (3-3). A particular focus of this formulation is to allow better estimation of truck flows in urban networks, using various size classes of trucks, and separating them from estimation of automobile O-D patterns.

The O-D estimation process is constructed using a probit-based stochastic model for network loading and user equilibrium. Using a stochastic model of user equilibrium (SUE) allows a relaxation of the common assumption that all drivers have complete and accurate knowledge of link travel costs everywhere in the network. The stochastic model includes errors in individual drivers' perceptions of those costs. Although the probit model of SUE is more demanding computationally than the logit-based model, it has important conceptual advantages with respect to the assumption about the distribution of errors and in reflecting covariance among path alternatives that share common links.

Sections 3.3-3.8 describe in detail an algorithm for solving the O-D estimation problem formulated in section 3.2. This algorithm borrows pieces from the work of several previous authors, but it represents a significant advance in capability for estimating multiclass O-D tables.

The following chapter describes an extensive set of experiments with the algorithm on a test network. These experiments illustrate the use of the algorithm, and also contribute insight into the relation between the amount and character of available data and the quality of the resulting O-D estimates.

Chapter 4

TESTS AND ANALYSIS

To test the concepts and solution algorithm, a series of experiments has been developed. These experiments are designed to test performance of the solution method under varying conditions and with varying types of data. Section 4.1 introduces the test network and the method to generate observation data for tests. Section 4.2 describes basic tests using a small three-vehicle-class set of O-D tables. In Section 4.3 the size of the O-D tables are expanded and additional tests for various types of available data are conducted. Section 4.3 summarizes the conclusions from the testing program.

4.1 Test Network

All the tests in this chapter are performed on the "Sioux Falls" (SF) network, shown in Figure 4-1. It originates from a representation of part of the street network in Sioux Falls, South Dakota. This network, first constructed and used by LeBlanc, *et al*. (1975), has since become a standard test network for many types of transportation network algorithms. The basic structure of the network has been retained from the original version used by LeBlanc, et al. (1975), but for the current testing, O-D tables and link characteristics have been created that enhance the network's usefulness as a test bed for the multiclass O-D estimation algorithm. By specifying an O-D table for each vehicle class and using those tables to create assigned link volumes, a test bed is created where different sets of volume data can be selected and the algorithm can be tested for its ability to reproduce the known O-D tables under varying conditions of available data. The known O-D values are not themselves used as data in any of the tests.

The tests are performed using three vehicle classes – automobiles, medium trucks and heavy trucks. Table 4-1 summarizes the vehicle-class parameters used for calculation of equivalent flow and class-specific costs for the multi-class SUE assignment.

4.2 Tests of Algorithm Validity and Effectiveness with Varying Data Availability

Initial tests are performed with a small set of four origin and destination zones (nodes 1, 7, 15 and 20), in which the general flow patterns for automobiles and the two truck classes are similar. Tables 4-2 through 4-4 specify the assumed O-D tables for the three vehicle classes. In this case, there are 4x3x3 = 36 unknown O-D volumes to be estimated, and if classified link counts are available for all links in the network, there is sufficient information (76x3 = 228 observations) to estimate the O-D tables quite accurately, even though not all the observations are independent because flow must be conserved at nodes within each vehicle class.

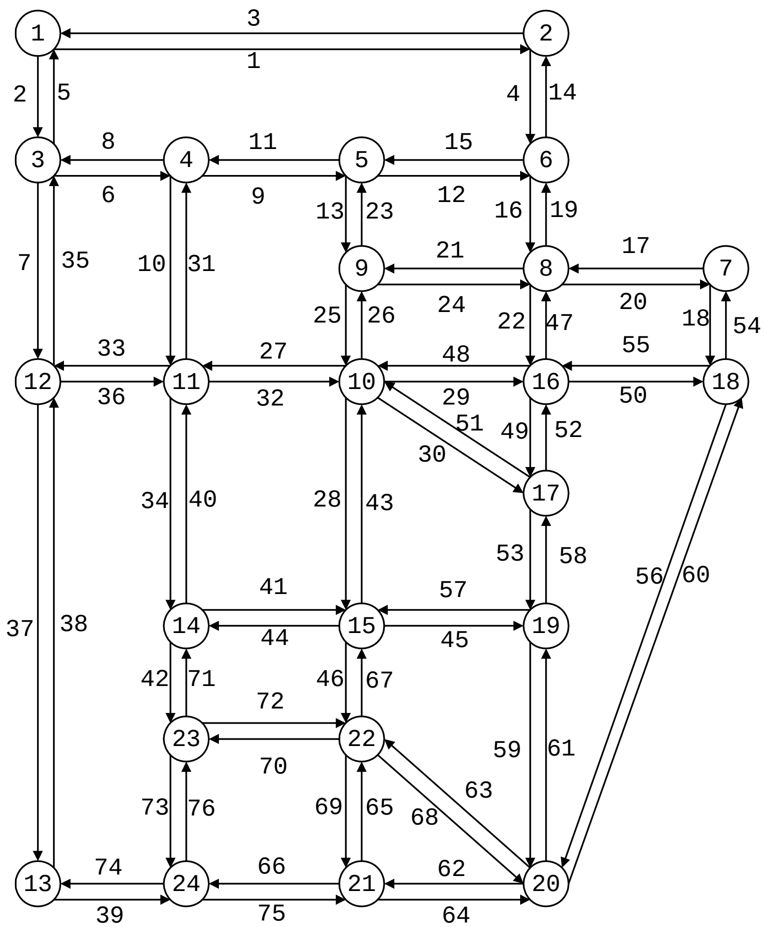

Figure 4 - 1. Sioux Falls network

Table 4 - 1. Vehicle class information in Sioux Falls network

Class number	Type	Cost of time Coefficient	Cost of distance Coefficient	Vehicle equivalents
1	Automobile	0.2	0.25	1
2	Medium truck	0.33	1	2
3	Heavy truck	0.5	1.5	3

Table 4 - 2. Four-zone O-D table for vehicle class 1 (veh/hr)

O\D	1	7	15	20
1	0	3109	839	891
7	3117	0	2462	1994
15	846	2456	0	1788
20	881	2010	1779	0

Table 4 - 3. Four-zone O-D table for vehicle class 2 (veh/hr)

O\D	1	7	15	20
1	0	215	61	57
7	214	0	163	141
15	58	161	0	123
20	64	131	110	0

Table 4 - 4. Four-zone O-D table for vehicle class 3 (veh/hr)

O\D	1	7	15	20
1	0	176	50	52
7	169	0	125	103
15	46	133	0	89
20	55	109	111	0

For each test run, the estimation error for each entry of each O-D table is measured and converted to a percentage of the actual value. A single O-D pair as judged to be estimated correctly if the estimation error is within $\pm 5\%$. The percentage of O-D pairs estimated correctly for each vehicle class and overall is one important performance measure for the algorithm. There is also concern with the total volume of trips presented by the accurately estimated O-D pairs. If the O-D volumes for the correctly estimated O-D pairs is small, that means a large proportion of traffic is estimated incorrectly, which is undesirable even if the method captures a high percentage of O-D pairs. Therefore, the percentage of O-D volume (classified or aggregated) that is estimated within the specified $\pm 5\%$ range is also an important performance measure.

When vehicle counts classified by vehicle class are available for all network links, the estimation accuracy can be summarized as shown in Figure 4-2, where the percentage errors for the 36 unknown O-D pairs are sorted from most negative (estimated volume less than actual) to largest (estimated volume greater than actual). 32 of the 36 unknown O-D volumes (89%) fall into the range between -5% and +5%, which accounts for 96% of O-D volume. Considered by the different vehicle classes, 91.7% of automobile, 91.7% of medium tracks, and 83.3% of heavy truck O-D pairs are estimated within the acceptable error, with corresponding volume of 96.2%, 95.9% and 91.6% respectively. This clearly indicates that the estimation algorithm operates successfully when sufficient data are available.

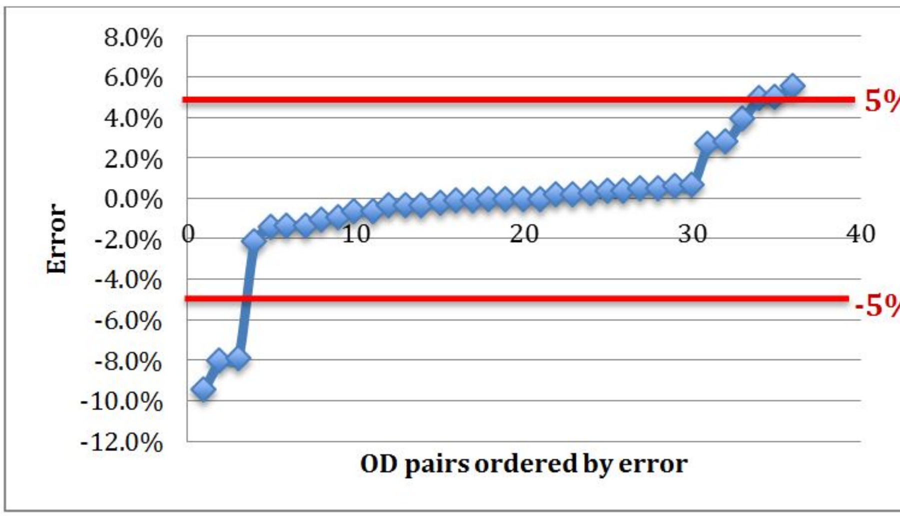

Figure 4-2. Sequenced error for 100% link coverage with classified vehicle counts

However, in practice it is unlikely that counts are available on all links. A set of tests was conducted using different percentages of links that covered by classified link count sensors, ranging from 10% to 100% in increments of 10%. Each case used 20 samples of randomly selected links except for the 100% case. The results are summarized graphically in Figure 4-3. For this network, reasonably accurate O-D estimates are obtained as long as the percentage of links with counts exceeds about 50%. If the percentage of links covered is below that value, the overall error of estimation increases dramatically. At a given level of link coverage, the total percentage of O-D volume estimated within ±5% error is bigger than the percentage of corresponding number of O-D pairs, which means that the solution estimates of large volume O-D pairs tend to be more accurate than estimates of small volume O-D pairs. Most of the O-D entries with largest percentage errors are in vehicle class 3 (heavy trucks), where the volumes are relatively small. Of course, on a small base volume a modest absolute error can yield a relatively large percentage error. However, these results do indicate that the truck O-D volumes, which are generally much smaller than automobile O-D volumes, are harder to estimate accurately.

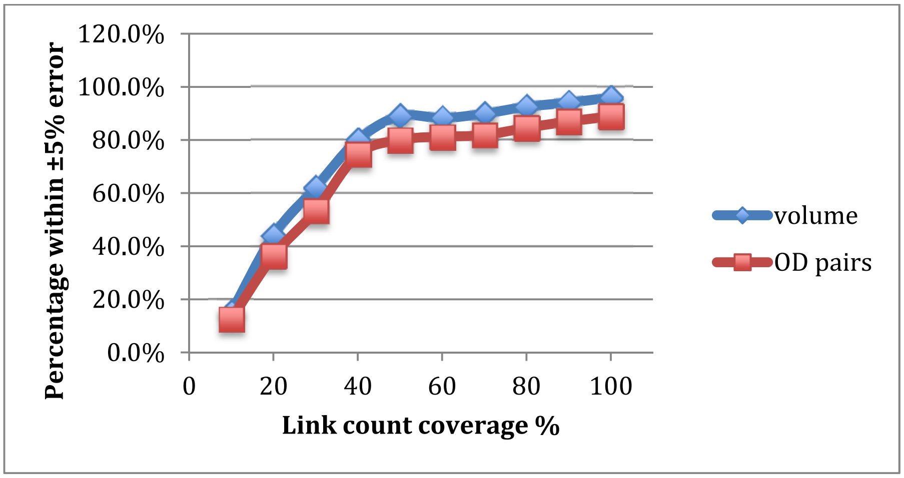

Figure 4-3. Percentage of O-D pairs and volume within 5% error under different link count coverage

In practice, it is also likely that not all available counts will contain vehicle classification information. In particular, single loop counters that are widely deployed in practice can only provide aggregate traffic counts. Collecting vehicle classification data is generally more expensive than collecting aggregate counts. In another set of tests, we examine how the accuracy of the multiclass O-D estimates is affected when only part of the link count sensors can provide classification information. The tested conditions are 100%, 75%, 50%, 25%, and 0% of links having classified count data. The 100% case corresponds to the initial experiment.

The results are illustrated in Figure 4 – . As the proportion of counts that include classification information decreases, the percentage of O-D entries estimated within 5% of actual values declines almost linearly, and this relationship is particularly true for the truck classes. The automobile class is more tolerant to the loss of vehicle classification data because most of the vehicles being counted are automobiles. These figures illustrate that it is very difficult to estimate truck O-D volumes with any accuracy when a significant fraction of the sensors are reporting only total vehicles counted.

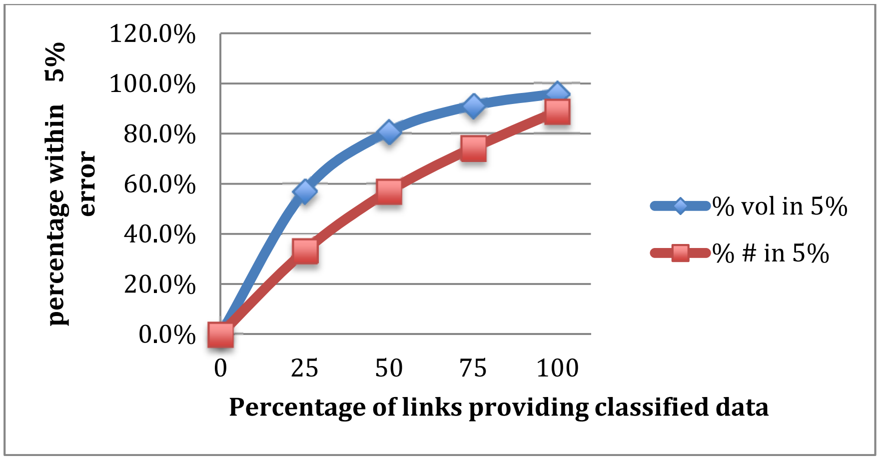

Figure 4 – 4. The percentage of O-D pairs and volume within 5% estimation error

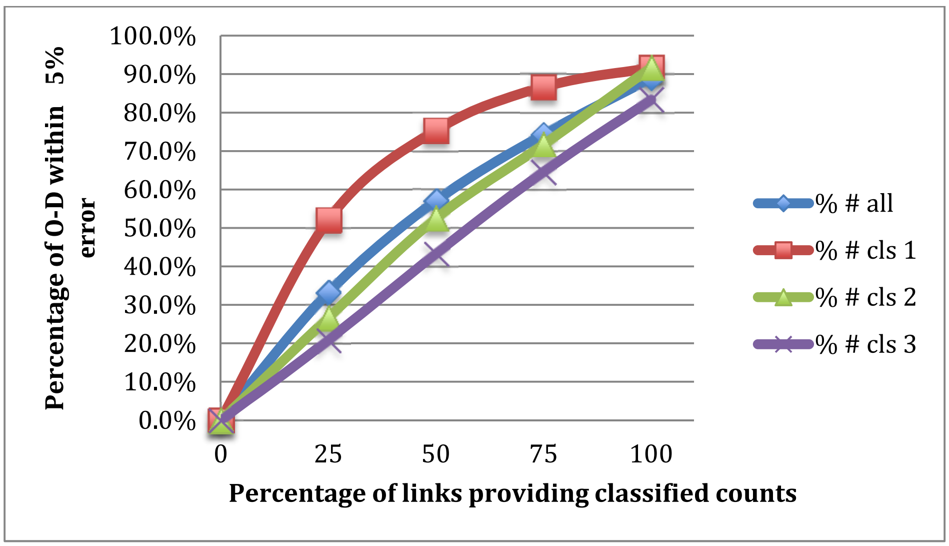

Figure 4 – 5. Percentage of O-D pairs within 5% error for each vehicle class

In addition, another set of tests explores the joint effect of losing some vehicle classification data and some link coverage. The tests relating to link coverage were repeated with only 50% of the available sensors able to provide classified counts. The rest are assumed to be single loop detectors that report only total vehicles. The results are summarized in Table 4-5, which can be compared to the graphical results in Figure 4-3**Error! Reference source not found.**. When only 50% of the sensors can provide classified count information, the quality of the estimations decreases for all vehicle classes, and especially for the two truck classes. Even with 80% coverage on network links, if only half of those counts contain vehicle classification information the number of truck O-D entries that are within the ±5% error range is only about 30%.

Table 4-5. Estimation errors of different level of link coverage with 50% of sensors providing classified link counts

link coverage	OD in 5%	OD vol. in 5%	ODs cls. 1	ODs cls. 2	ODs cls. 3
100%	57.1%	80.7%	75.4%	52.5%	43.3%
90%	52.4%	75.6%	70.4%	46.3%	40.4%
80%	43.9%	72.3%	69.2%	31.7%	30.8%
70%	44.4%	68.9%	65.4%	35.8%	32.1%
60%	43.3%	64.5%	57.9%	38.3%	33.8%
50%	37.2%	59.4%	55.8%	28.8%	27.1%
40%	26.7%	46.6%	39.6%	19.2%	21.3%
30%	17.8%	33.9%	27.5%	11.3%	14.6%
20%	10.6%	18.1%	15.0%	8.3%	8.3%
10%	7.9%	7.8%	5.4%	6.7%	11.7%

These experiments reinforce the importance of having vehicle classification data as part of the traffic counts. If it is desired to do multiclass O-D estimation, it is more important to have classification data from whatever links are available than to have full coverage of the network links.

If vehicle classification data is important, but expensive to collect, it is useful to ask if partial classification data from much cheaper dual loop detectors would be sufficient. They cannot separate all vehicle classes, but can distinguish an automobile from a truck. To explore the potential for dual loop detector data, we tested a situation with 100% link count coverage, where 100%, 50%, and 0% of them are dual loop detectors. The remaining counts are assumed to be total vehicle counts from single loop sensors.

The test results are organized in Table 4-6. Without classified link counts, the estimation of truck O-D volumes is completely ineffective. Even if all sensors can differentiate between automobiles and trucks (i.e. 100links_100dual case), the estimates of truck O-D volumes are nearly all incorrect. It is preferable to have 50% classification counts than to have 100% dual loop counts. When only single loop detectors exist, even with 100% link coverage, none of the O-D entries (automobile or truck) is estimated usefully. Interestingly, even the automobile O-D estimates are degraded by substituting dual loop counts for classification counts. This is the case even though the automobile observations are the same in each case. The confusion induced in trying to sort out the truck classes from one another without actual classification data carries over into the automobile estimates as well. We conclude that using dual loop detectors to substitute for classification counts in estimating O-D tables with more than one truck class is not likely to prove useful.

Table 4-6. Effects of using dual loop counts rather than classification counts.

	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
100links_100clsfy	88.9%	96.0%	91.7%	91.7%	83.3%
100links_100dual	30.6%	82.7%	83.3%	0.0%	8.3%
100links_50clsfy	57.1%	80.7%	75.4%	52.5%	43.3%
100links_50dual	22.6%	70.1%	65.0%	1.7%	1.3%
100links_100single	0.0%	0.0%	0.0%	0.0%	0.0%

In actual cities, not all the links are covered by sensors and most deployed sensors are single loop detectors. Even in cities noted for effective traffic management, like Rochester, NY, only 20% of links in the central city area are covered by sensors, all of which are single loop detectors. The results from the earlier sets of tests offer little hope of doing effective multiclass O-D estimation (or even single class estimation) with such sparse data, but as an explicit test case, we have postulated a situation where 20% of links have counts, and 25% of those counts are from dual loop sensors. We compare that with a case where all the counts are from single loop sensors.

The test results are summarized in Table 4-7. Comparing the last two rows of Table 4-7, we conclude that even if 25% of the sensors are upgraded to dual loop sensors, few reliable estimates will be derived, particularly for trucks. We have also tested two more extensive updates of typical current counter data – upgrading either 50% or 100% of the current link counters to provide vehicle classification data. Upgrading 50% of the counters (i.e., having classification data on 10% of total network links) to provide such data has very modest benefits. Some O-D entries are estimated more accurately, but the overall results are still quite poor. Even if all the sensors were upgraded to provide classified link counts (the 20% case in Figure 4-3), accuracy of the results is strongly limited by the sparse coverage of the data across the network.

Table 4-7. Effects of partial link coverage and partial classification counts.

	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
20links_100clsfy*	36.4%	44.0%	36.7%	36.3%	36.3%
20links_50clsfy	10.6%	18.1%	15.0%	8.3%	8.3%
20links_25 dual	9.2%	22.7%	19.2%	2.1%	6.3%
20links_0dual	2.8%	3.5%	3.8%	0.4%	4.2%

* Note: the notation of the cases: '20links' means 20% of total links in the network are covered by sensors. '100clsfy' means 100% of the sensors can classify three vehicle classes.

Estimating multiclass O-D tables with the level of data typically available currently is largely a futile exercise. More sensors capable of providing vehicle classification information are needed, and there is also potentially an important role for data types other than link counts in order to estimate multiclass O-D tables.

4.3 Tests with an Expanded Set of O-D Pairs

A second set of O-D tables for the same network contains seven origin and destination zones (nodes 1, 6, 7, 10, 13, 15 and 20) and has considerably different flow patterns for automobiles and trucks (see Tables 4-8 through 4-10).

Table 4-8. Seven-zone O-D table for vehicle class 1 (veh/hr)

O\D	1	6	7	10	13	15	20
1	0	491	440	1070	532	839	359
6	491	0	306	269	224	266	264
7	442	308	0	1348	90	723	90
10	1064	267	1343	0	532	533	794
13	526	221	90	537	0	708	91
15	846	267	714	534	719	0	1069
20	355	267	88	807	89	1071	0

Table 4-9. Seven-zone O-D table for vehicle class2 (veh/hr)

O\D	1	6	7	10	13	15	20
1	0	14	37	121	69	24	19
6	25	0	19	37	16	7	10
7	30	19	0	50	37	7	18
10	116	45	62	0	116	38	49
13	68	23	30	113	0	27	25
15	19	7	14	33	25	0	15
20	19	3	18	33	28	3	0

Table 4-10. Seven-zone O-D table for vehicle class 3 (veh/hr)

O\D	1	6	7	10	13	15	20
1	0	42	53	121	41	48	39
6	38	0	8	19	13	6	8
7	68	20	0	37	18	10	18
10	115	34	40	0	26	28	23
13	33	15	8	36	0	18	14
15	27	7	17	32	8	0	14
20	32	13	21	23	5	8	0

With this larger set of unknown O-D volumes (7x6x3 = 126 unknowns), even with full classification data on all links, the outcome is quite poor. Only 12.7% of O-D pairs are estimated within 5%, which includes about 18.8% of the total traffic volume. These results are summarized in Table 4-11, which includes the values from the four-zone case for comparison. These results emphasize the under-specification of the O-D estimation problem when only link counts are used as data.

If link counts alone (even with 100% of links covered by classification counts) are insufficient, it is reasonable to ask whether augmenting the link counts by turning movements at intersections can improve the results. As noted in Chapter 2, most of the literature on O-D estimation has paid little attention to turning movements because the partial path information they provide is not important under deterministic user equilibrium assumptions, where path flows are not uniquely identified. However, SUE assignments do identify path probabilities, and turning movements may be of greater value. Turning movements are typically collected manually, but they can be provided by surveillance cameras and there has been some work on reconstructing them from inbound and outbound loop sensors at intersections. The tests in this section add turning movement data from 2, 4, and 6 non-centroid nodes to the original link count information. The two-intersection tests include turning movements at nodes 11 and 16; the four-intersection test adds nodes 3 and 22 to those; and the six-intersection test adds nodes 8 and 19.

Table 4-11. Estimation results when classified link counts with full link coverage available

	4 x 4 O-D	7 x 7 O-D
Maximum error	5.5%	234.7%
Minimum error	-9.4%	-100.0%
Number of OD pairs within 5%	88.9%	12.7%
OD volumes within 5%	96.0%	18.8%
OD pairs within 5% for vehicle class 1	91.7%	11.9%
OD pairs within 5% for vehicle class 2	91.7%	16.7%
OD pairs within 5% for vehicle class 3	83.3%	9.5%
OD volumes within 5% for vehicle class 1	96.2%	18.8%
OD volumes within 5% for vehicle class 2	95.9%	20.0%
OD volumes within 5% for vehicle class 3	91.6%	17.3%

Three types of turning movement data are tested. Classified turning movements can provide counts for all three vehicle classes. Dual loop turning movements are assumed to be reconstructed from dual loop counters on entry to and exit from the intersection. These counts differentiate automobiles from trucks, but group both truck classes together, as with the dual loop link counts. Total vehicle turning movements give only total counts of vehicle movements. In addition, as a base case, we include the case where no turning movements are available.

We combined each type of the turning movement counts to three kinds of link counts: 100% link coverage with all 100% classified link counts; 100% link coverage with all 100% being dual loop detectors; and 80% of links covered with all assumed to be classified link counts. Each of these three cases is tested with different numbers of intersections considered. This creates a total of 36 experiments, including all combinations of the three experimental factors.

Table 4-12 summarizes the results for six intersections with turning movements. Parallel sets of results for four and two intersections are shown in Tables 4-13 and 1-14. The base case results (no turning movements) are provided for comparison. Availability of turning movements obviously improves the estimation result as the information provided from observations increases. Especially when no classified information from link counts is available, classified turning movements can help to improve the truck estimation very substantially.

Table 4-12. Estimation supplemented by different turning movement data at 6 intersections

7x7 OD	6 intersections	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
100% link with all classified link counts	no turning	12.7%	18.8%	11.9%	16.7%	9.5%
	classified turning	35.7%	53.5%	45.2%	31.0%	31.0%
	dual turning	36.5%	54.2%	42.9%	35.7%	31.0%
	single turning	27.0%	52.8%	47.6%	16.7%	16.7%
80% link with all classified link counts	no turning	14.4%	26.7%	20.2%	12.0%	10.8%
	classified turning	35.0%	49.1%	39.6%	34.8%	30.7%
	dual turning	33.4%	54.8%	45.1%	30.1%	25.0%
	single turning	18.1%	36.8%	33.6%	11.1%	9.6%
100% link with all dual loop counts	no turning	9.5%	33.1%	23.8%	2.4%	2.4%
	classified turning	39.7%	58.3%	45.2%	45.2%	28.6%
	dual turning	12.7%	43.5%	28.6%	0.0%	9.5%
	single turning	7.1%	32.1%	21.4%	0.0%	0.0%

For example, before adding any turning movements, with 100% link coverage by dual loop sensors, only 2.4% of truck O-D pairs are estimated within the specified ±5% range. After adding classified turning movement counts at six intersections, the accuracy rate is raised to 45.2% for medium trucks and 28.6% for heavy trucks. The accuracy rate for automobiles has been doubled. Overall, the number of O-D pairs that have acceptable errors has increased from 9.5% to almost 40% with classified turning movement counts. When classification information is not available in the link count data, classified turning movements can improve O-D estimation dramatically, especially for trucks. However, dual loop turning counts and total vehicle turning movements are much less effective.

Table 4-13. Estimation supplemented by different turning movement data at 4 intersections

7x7 OD	4 intersections	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
100% link with all classified link counts	no turning	12.7%	18.8%	11.9%	16.7%	9.5%
	classified turning	42.1%	57.1%	52.4%	42.9%	31.0%
	dual turning	41.3%	60.5%	50.0%	40.5%	33.3%
	single turning	27.0%	51.7%	47.6%	16.7%	16.7%
80% link with all classified link counts	no turning	14.4%	26.7%	20.2%	12.0%	10.8%
	classified turning	31.9%	47.1%	40.1%	28.2%	27.3%
	dual turning	30.1%	47.4%	39.8%	26.9%	23.6%
	single turning	16.6%	33.4%	30.2%	10.8%	8.8%
100% link with all dual loop counts	no turning	9.5%	33.1%	23.8%	2.4%	2.4%
	classified turning	29.4%	52.6%	42.9%	26.2%	19.0%
	dual turning	11.9%	39.6%	31.0%	2.4%	2.4%
	single turning	9.5%	31.4%	23.8%	2.4%	2.4%

Table 4-14. Estimation supplemented by different turning movement data at 2 intersections

7x7 OD	2 intersections	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
100% link with all classified link counts	no turning	12.7%	18.8%	11.9%	16.7%	9.5%
	classified turning	27.8%	48.6%	38.1%	23.8%	21.4%
	dual turning	30.2%	48.5%	42.9%	23.8%	23.8%
	single turning	28.6%	52.2%	47.6%	23.8%	14.3%
80% link with all classified link counts	no turning	14.4%	26.7%	20.2%	12.0%	10.8%
	classified turning	25.5%	41.0%	35.0%	19.9%	21.7%
	dual turning	20.7%	39.0%	32.5%	14.8%	14.9%
	single turning	14.0%	27.1%	23.3%	10.4%	8.5%
100% link with all dual loop counts	no turning	9.5%	33.1%	23.8%	2.4%	2.4%
	classified turning	16.7%	34.9%	23.8%	11.9%	14.3%
	dual turning	7.9%	32.3%	23.8%	0.0%	0.0%
	single turning	7.9%	25.8%	19.0%	2.4%	2.4%

Even if we had classified link counts on all the links, adding turning movements still increases the accuracy rate from 12.7% to around 36%. In this case, dual loop turning movements produce a similar improvement. When the classification information is available from link counts, classified turning movements can be substituted by cheaper dual turning movements and have similar overall O-D estimation accuracy. This conclusion also holds for the 80% link coverage cases.

When only total vehicle turning movements are available, the primary effect is on estimates of automobile (class 1) O-D volumes. If classification counts are available on links, addition of total turning movements at intersections improves the automobile estimation accuracy, although it provides little benefit for the truck classes.

All of the above conclusions also hold for supplementing by 4 intersections and 2 intersections. However, as the number of intersections providing turning movement is reduced, the result is less dramatic.

Four important conclusions emerge from these experiments. First, turning movement data can be quite useful in improving the accuracy of multiclass O-D estimates. Second, classification counts at intersections and along links can be substitutes for one another, as long as classification data is available from some source. Third, for the truck classes, total vehicle turning movements are not very useful, even if vehicle classification data is available from the link counts. Some distinction between automobiles and trucks is necessary, even if only from dual loop counters at the intersections. Finally, having more intersections with turning movement data is better, although even a few intersections can be useful.

All the previous tests are performed based on error-free observations. However, in reality, all the observations are subjected to a certain level of error depending on the types of sensors. We are interested in how the solution method performs with observation errors. Using current technology, the error of dual loop link counts is around 4%, and data for classified link counts is approximately 10% (Yu, *et al*. 2010). For turning movements, the current technology can provide unclassified counts at an error of approximately 8% (Yi, *et al*. 2010). For the experiments done here, both dual and classified turning movements are assumed to have 10% errors. The error of the sensors is assumed to be the coefficient of variation of a normal distribution with the mean of the true traffic volume on the link or turning path. We selected four situations from the tests on turning movements that have relatively higher accuracy and introduce the error to the observations.

A summary of the results is shown in Table 4-15. In all four test cases, there is marked deterioration in the quality of the estimated O-D tables when the observed data contains errors of the magnitude that might normally be expected in practice. This is not unexpected, but these results emphasize two important aspects of the multiclass O-D estimation problem. First, it is important to reduce the errors in the observed data whenever possible. This may mean using better counting technology, or doing more filtering of the data before it is used. Second, in the presence of errors it is necessary to have far more observations to support O-D estimation than would be necessary in an error-free environment. This has implications for sensor location, which includes consideration of both how many sensors, and where they should be located.

Table 4-15. Comparison of estimation with error and no error introduced

	Error test	ODs in 5%	OD volume in 5%	ODs in 5% for cls 1	ODs in 5% for cls 2	ODs in 5% for cls 3
100% link covered by all classified link counts with dual turning in 4 intersections	no error	41.3%	60.5%	50.0%	40.5%	33.3%
	with error	11.1%	23.9%	16.7%	14.3%	2.4%
100% link covered by all dual loop counts with classified turning in 6 intersections	no error	39.7%	58.3%	45.2%	45.2%	28.6%
	with error	15.1%	7.1%	9.5%	19.0%	16.7%
80% link covered by all classified link counts with classified turning in 6 intersections	no error	35.0%	49.1%	39.6%	34.8%	30.7%
	with error	10.1%	15.7%	13.0%	7.9%	9.5%
80% link covered by all classified link counts with dual turning in 6 intersections	no error	33.4%	54.8%	45.1%	30.1%	25.0%
	with error	9.2%	14.9%	11.7%	8.3%	7.5%

4.4 Conclusions

In this chapter, a series of experiments have been carried out on a test network to address several questions regarding the estimation of multiclass O-D tables. The results of these experiments verify the ability of the method developed here to successfully estimate O-D tables when sufficient data are available, and also lead to important conclusions about the likely success of multi-class O-D estimation under more typical scenarios of data availability. These conclusions are:

1) Without classified link counts, the estimation of truck O-D volumes is completely ineffective. Even if all sensors can differentiate between automobile and trucks (i.e. dual loop counters), the estimates of truck O-D volumes are nearly all incorrect. It is preferable to have 50% classification counts than to have 100% dual loop counts. Using dual loop detectors to substitute for classification counts in estimating O-D tables with more than one truck class is not likely to prove useful.
2) When only single loop detectors exist, even with 100% link coverage, none of the multiclass O-D entries (automobile or truck) is estimated usefully. This is important because single loop counters are currently the dominant source of traffic counts in most urban areas. Moreover, the link coverage in practice is much less than 100%, and under these conditions, it is likely to be a futile exercise to attempt estimation of multiclass O-D tables.
3) Adding turning movement data to link counts is an effective way of increasing the quality of O-D estimates. When only total vehicle turning movements are available, the primary effect is on estimates of automobile (class 1) O-D volumes. If classification counts are available on links, addition of total turning movements at intersections improves the automobile estimation accuracy, although it provides little benefit for the truck classes.
4) When classification information is not available in the link count data, classified turning movements can improve estimation of truck O-D tables dramatically.
5) If vehicle classification counts are available from the link data, classified turning movements can be substituted by cheaper dual turning movements and have similar overall O-D estimation accuracy.
6) There is marked deterioration in the quality of the estimated O-D tables when the observed data contains errors of the magnitude that might normally be expected in practice. This is not unexpected, but this emphasizes two important aspects of the multiclass O-D estimation problem. First, it is important to reduce the errors in the observed data whenever possible. This may mean using better counting technology, or doing more filtering of the data before it

is used. Second, in the presence of errors it is necessary to have far more observations to support O-D estimation than would be necessary in an error-free environment. This has implications for sensor location, which includes consideration of both how many sensors, and where they should be located.

All of these conclusions are based on use of data solely from traffic monitoring. In addition to such data, it may be useful to augment the O-D estimation process (particularly for trucks) with data on economic activity in zones. It is also important to test the overall process on a much larger and more realistic network. This is the subject of the case study in the following chapter.

Chapter 5

A CASE STUDY IN ROCKLAND COUNTY, NEW YORK

Rockland County is on the west side of the Hudson River, just north of New York City, located as shown in Figure 5-1. The county is roughly triangular in shape, covering 199 square miles with a total population of just over 310,000. Figure 5-2 shows additional detail about the county, including some of the main highways. The county contains a portion of the New York State Thruway (I-87) leading to the Tappan Zee Bridge and is roughly bisected by the Palisades Interstate Parkway, running north-south. It also contains the connections between I-87 and two other major facilities going south into New Jersey – the Garden State Parkway and I-287. The Palisades Interstate Parkway and the Garden State Parkway are closed to commercial traffic. However, in addition to I-87 and I-287, portions of several state highways in the county are heavily used truck routes.

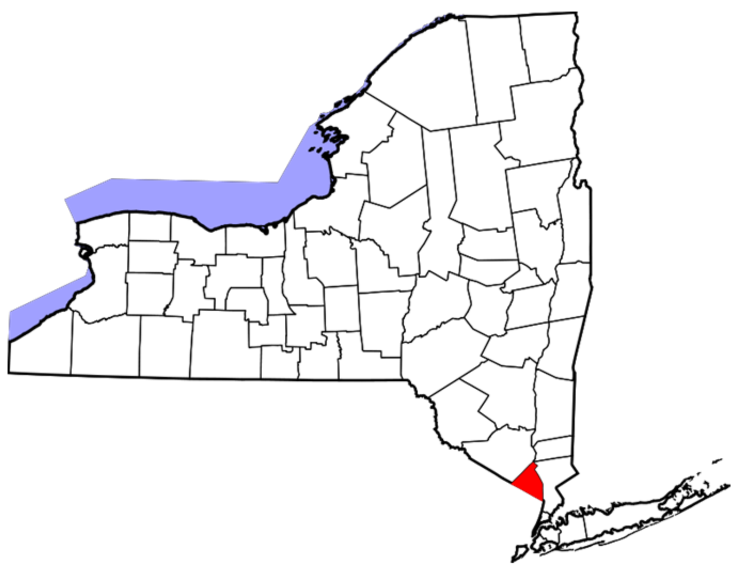

Figure 5-1. Location of Rockland County, New York.

The principal sources of data for this test case are average daily vehicle counts collected by the state Department of Transportation, and estimates of truck trip generation based on land use. The experiments with this test network are designed to answer two basic questions:

1) Does the methodology developed in this project scale reasonably to a much larger network than used in Chapter 4, with a much more complex structure and variety of facilities?

2) If basic count data is augmented by commonly available planning data (e.g., land use data) and generic truck trip generation estimates based on that data (from earlier studies in other locations), how are the estimated O-D tables affected?

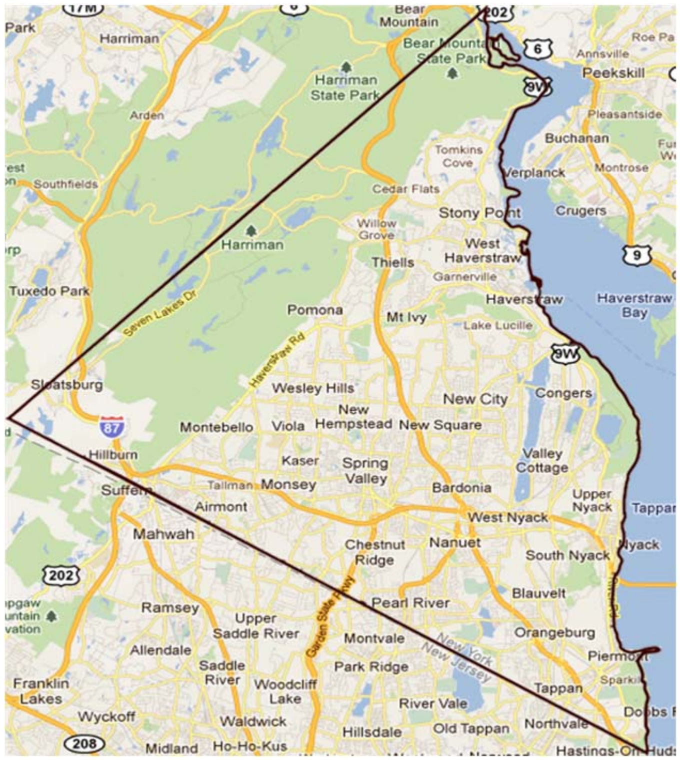

Figure 5-2. Rockland County, New York.

5.1 Network Construction

The network representation used contains 25 traffic analysis zones (TAZs), where trips originate and terminate. 14 of these zones are internal and the remaining 11 represent the entry/exit points to the county from adjoining areas, including the Tappan Zee Bridge. There are thus 600 O-D pairs for each vehicle class, although some of these (origins and destinations for trucks at the external zones representing the parkways) are required to have zero trips. Table 5-1 summarizes the location of the TAZs.

Table 1-1 Traffic Analysis Zones

TAZ code	Description
301	External zone on the north of the Rockland county, linked by highway 9W
302	External zone on the northwest of the Rockland county, linked by Palisades Interstate Parkway
303	Local zone representing Stony Point
304	Local zone representing Haverstraw town
305	Local zone representing Haverstraw village
306	Local zone representing New Hempstead
307	Local zone representing New Square
308	Local zone representing Clarkstown
309	Local zone representing Spring valley
310	Local zone representing Nyack
311	External zone on the east of Rockland County, linked by Tappan Zee Bridge
312	External zone on the south of Rockland County, linked by I- 287
313	Local zone representing Suffern
314	External zone on the south of Rockland county, representing NJ local
315	Local zone representing Airmont
316	Local zone representing Chestnut Ridge
317	External zone on the south of Rockland County, linked by Garden State parkways
318	External zone on the south of Rockland county, representing NJ local
319	Local zone representing Orangetown
320	External zone on the south of Rockland county, representing NJ local
321	External zone on the south of Rockland County, linked by NJ Palisades Interstate Parkway
322	Local zone representing Sloatsburg
323	External zone on the west of Rockland county, representing Orange county local
324	External zone on the west of Rockland County, linked by I-87
325	Local zone representing West Haverstraw village

The network includes 277 nodes and 771 links (including centroid connectors to the TAZs). The network is designed to represent the designated state highways in the county, with other links added as necessary to make connections. The overall network is shown in Figure 5-3.

Each link has attributes of length, number of lanes, free-flow speed and overall capacity. The capacity values are based on the functional classes of the links, using the class definitions in Table 5-2 (from NYSDOT). Centroid connectors for the internal TAZs are assumed to be 0.5 mile in length and have a fixed speed of 25 mph. An delay function (Akçelik , 1991) of the form shown in eq. 3-6 is used for the travel time calculations on links.

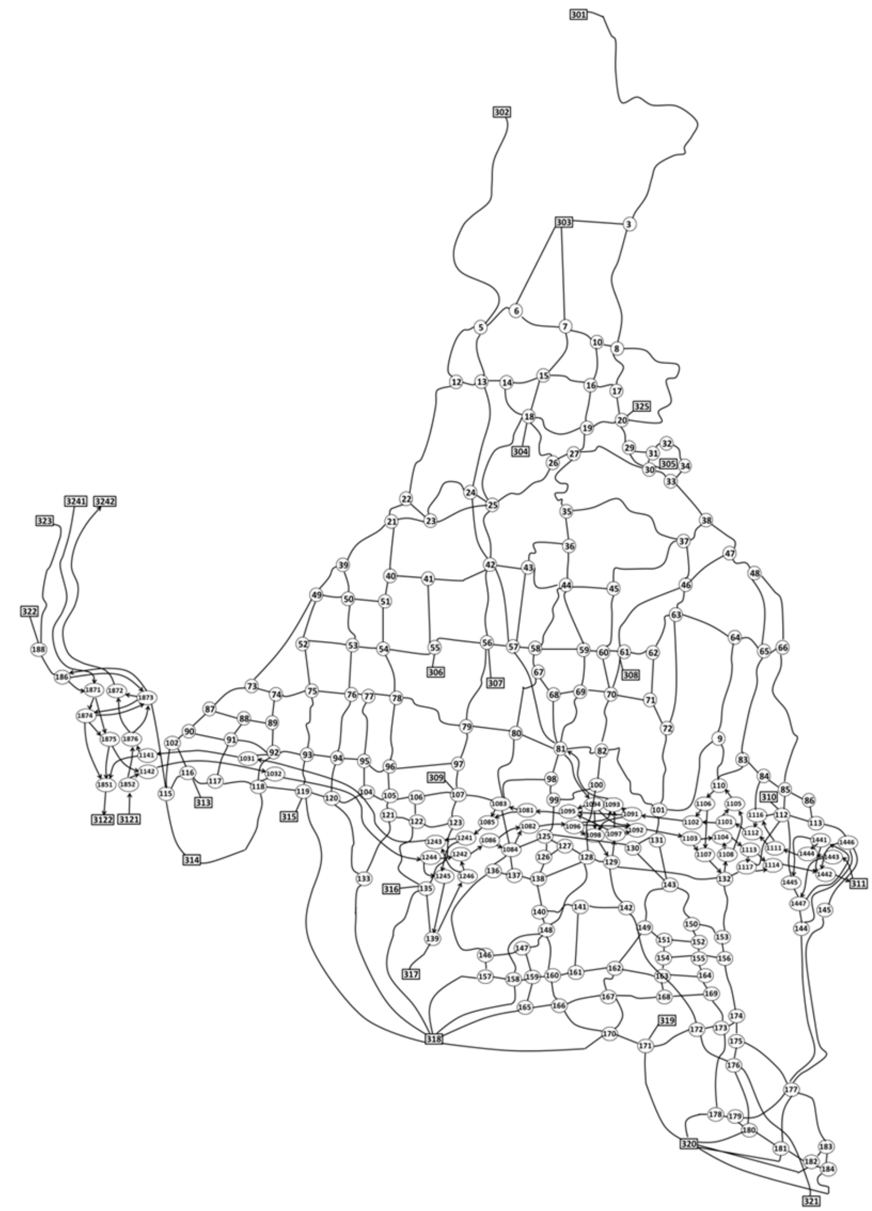

Figure 5-3. Rockland County network.

Table 5-2. Road classes included in the Rockland model.

Functional Classification Code	Description
11	Urban Interstate
12	Urban other Freeway and Expressway
14	Urban Principal Arterial
16	Urban Minor Arterial
17	Urban Major Collector
20	Ramp (speed limit below 45 mph)
21	Ramp (speed limit greater or equal to 45 mph)
999	Centroid Connector

5.2 Constructing Link Count Observations

NYSDOT provides average annual daily traffic (AADT) data for segments of the state highways, U.S highways, and the Interstate routes in Rockland County. These AADT values are not all from the same year, but are relatively consistent, and they have been used without correction for the various years. The case study is focused on an AM peak hour, so the AADT values are converted into directional hourly volumes (DHV) using eq. 5-1:

$$DHV = (AADT)(K)(D) \tag{5-1}$$

where: K= proportion of AADT occurring in the peak hour
D = proportion of peak-hour traffic in the peak direction.

For this network, the AM peak direction is southeast. Links whose direction is north or west are assumed to be in the off-peak direction, and the factor (1-*D*) is used instead. The K-factors used are taken from the 2010 Highway Capacity Manual, and shown in Table 5-3. The directional factors depend on facility type, and are also taken from the 2010 HCM, as shown in Table 5-4. The HCM facility types have been mapped into the NYSDOT functional classes for use in this network.

Table 5-3. K-Factors for different AADT [HCM 2010]

AADT	Average K -Factor
0-2,500	0.151
2,500 – 5,000	0.136
5,000 – 10,000	0.118
10,000 – 20,000	0.116
20,000 – 50,000	0.107
50,000 – 100,000	0.091
100,000 – 200,000	0.082
> 200,000	0.067

Table 5-4. Directional distribution [HCM 2010]

Roadway Type	D-Factor
Rural-intercity	0.59
Rural-recreational and intercity	0.64
Suburban circumferential	0.52
Suburban radial	0.60
Urban radial	0.70
Intraurban	0.51

The AADT values do not separate automobiles from trucks, but NYSDOT publishes an average "% trucks" for each functional class. These percentages have been used to estimate truck traffic on each link. Because this method for estimating truck volumes is relatively crude and does not distinguish among different classes of trucks, the analysis here is based on only two vehicle classes – automobiles and trucks. On the Palisades Interstate Parkway and the connectors to the Garden State Parkway, truck volumes are set to zero.

5.3 Using Employment Data to Estimate Truck Trip Rates

One of the objectives in the Rockland County test case is to experiment with using socio-economic data to augment traffic data for estimating truck O-D tables. One approach is to use employment data within census areas to estimate the number of truck trip ends (origins and destinations) within that area. The Southern California Association of Governments (SCAG) has developed estimates of trip rates related to employment in seven different industries as well as to numbers of households in an area (described in U.S. Federal Highway Administration, 2007). Although their estimates were created for use in the Los Angeles / Long Beach area, we are testing whether such estimates may be useful in other areas as well. These rates, which separate light, medium and heavy trucks, are shown in Table 5-5.

Table 5-5. Truck trip rates estimated based on employment and numbers of households in economic areas.

	Outbound				Inbound			
	LH	MH	HH	Subtotal	LH	MH	HH	Subtotal
Households	0.0390	0.0087	0.0023	0.0500	0.0390	0.0087	0.0023	0.0500
AMC	0.0513	0.0836	0.0569	0.1919	0.0513	0.0836	0.0569	0.1919
Retail	0.0605	0.0962	0.0359	0.1925	0.0605	0.0962	0.0359	0.1925
Government	0.0080	0.0022	0.0430	0.0533	0.0080	0.0022	0.0430	0.0533
Manufacturing	0.0353	0.0575	0.0391	0.1319	0.0353	0.0575	0.0391	0.1319
Transportation	0.2043	0.0457	0.1578	0.4078	0.2043	0.0457	0.1578	0.4078
Wholesale	0.0393	0.0650	0.0633	0.1677	0.0393	0.0650	0.0633	0.1677
Service	0.0091	0.0141	0.0030	0.0262	0.0091	0.0141	0.0030	0.0262

Notes:
LH = light-heavy (8,501-14,000 lb GVW)
MH = medium-heavy (14,001-33,000 lb GVW)
HH= heavy-heavy (>33,000 GVW)
AMC = Agriculture/Mining/Construction
*Household trip rates are trips per household; all other trip rates are trips per employee

To use these trip rates, it is necessary to estimate employment and numbers of households within defined zones. Employment data is from the 2007 U.S. Economic Census (U.S. Census Bureau, 2009) . The census divides Rockland County into 14 different economic areas, as shown in Figure 5-4. For the current purposes, data for New Square and Waverly Hills has been combined into Ramapo Town, which surrounds these two small areas. In addition, the census also contains data for the number of establishments and employees for various industries in these areas. However, in some cases the number of employees is given as a range of employment, and we have used a middle value from that range. Table 5-6 gives the estimates of number of employees by industry group for each area.

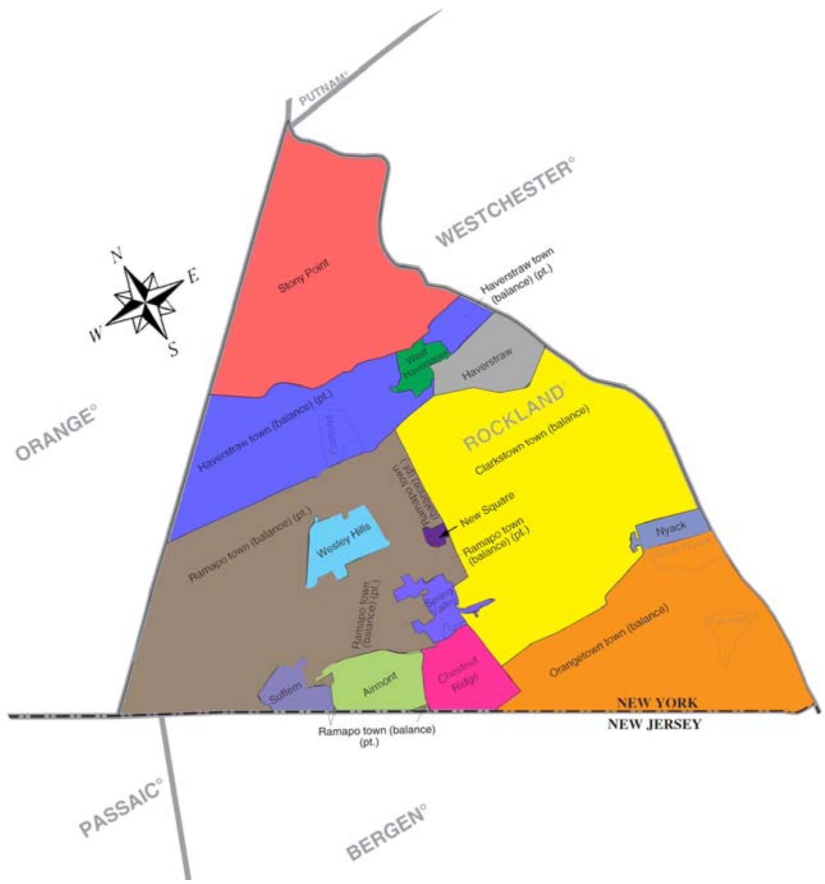

Figure 5-4. Economic areas in Rockland County, New York, from the 2007 Economic Census.

The industry categories defined in the SCAG rates do not match exactly the categories used in the Economic Census. The Retail, Manufacturing and Wholesale industry categories map directly to the comparable employment categories used in the trip rate model. The census data for Information, Real

Estate, and Arts, Entertainment, and Recreation are not considered, since these three industry categories produce negligible truck generation and destinations. The remaining industry categories are all mapped to the employment category Services for use in the trip rate model.

Table 5-6. Estimate of employees by industry group and area within Rockland County.

	Retail	Manufacturing	Wholesale	Information	Real estate	Prof, sci, tech srvs	Admin, support, wast mang srvs	Education	Health care	Arts, ent, recr	Accom, food	Other srvs
Stony Point	245	445	66	60	45	89	287	10	175	60	220	63
Haverstraw town	307	62	60	11	56	62	115	0	1339	18	197	138
West Haverstraw village	340	37	44	11	38	14	60	11	750	60	97	40
Haverstraw village	73	219	60	60	9	60	53	11	175	60	124	105
Ramapo town	1113	357	587	244	473	808	876	180	2791	205	875	614
Suffern	429	750	183	60	58	355	346	60	2544	60	561	86
Airmont	258	99	95	60	65	52	47	12	302	12	94	63
Spring Valley	1310	143	264	219	135	119	133	60	745	11	186	164
Clarkstown town	8503	1467	1142	1131	578	2139	1758	276	6589	747	3024	1184
Nyack village	166	181	90	76	114	198	80	12	1884	172	485	120
Orangetown town	964	5654	2777	1750	164	964	494	60	3270	401	1371	309
Chestnut Ridge	414	840	223	11	17	254	294	12	445	30	29	133

From the Census data for the county, we obtain the population in each of the economic areas, in addition to the average persons per household. We used these values to estimate the number of households in each economic area, as shown in Table 5-7.

We then construct estimates for daily truck trip origins and destinations by economic area. The destination (entering) values are summarized in Tables 5-8. Origins are identical numbers for each area. Because the analysis in this case is based on aggregating all truck classes into one, the values used in the O-D estimation process are the total values for each area shown in Table 5-8. These daily values are then factored to peak hour values and assigned as origin and destination totals for trucks in the zones corresponding to the economic areas within the county.

Table 5-7. Estimated households by economic area for use in the SCAG trip rate model.

Economic Place	Population	Number of Households
Stony Point	14244	4686
Haverstraw town	33811	11122
West Haverstraw village	10295	3387
Haverstraw village	10117	3328
Ramapo town	118377	38940
Suffern	11006	3620
Airmont	7799	2565
Spring Valley	25464	8376
Clarkstown town	82082	27001
Nyack village	6737	2216
Orangetown town	47711	15694
Chestnut Ridge	7829	2575

(Note: average persons per household = 3.04)

Table 5-8. Estimates of entering daily truck trip ends by economic area.

Economic Place	Entering			
	Light	Medium	Heavy	Total
Stony Point	224	106	44	374
Haverstraw town	474	160	48	682
West Haverstraw village	164	81	27	272
Haverstraw village	149	60	24	233
Ramapo town	1678	591	199	2468
Suffern	237	183	76	496
Airmont	128	67	27	222
Spring Valley	434	244	93	771
Clarkstown town	1800	1423	542	3765
Nyack village	132	91	32	255
Orangetown town	1038	826	487	2351
Chesnut Ridge	175	141	71	387

5.4 Experiments and Results

Two computational experiments are done for the Rockland County network. The first is estimation of a two-class (autos and trucks) O-D table for the 25-zone network based only on the most available data – link counts constructed from the AADT values maintained for the state highway system. This experiment accomplishes two objectives: it demonstrates the efficacy of estimating multi-class O-D tables on a realistic network, and it establishes a baseline result constructed with only the most basic data available. The second experiment adds the origin and destination totals for truck trips in the internal zones. This allows evaluation of the added information represented by the economic activity data.

For Rockland County, there is not an available truck O-D matrix to use as a standard for evaluation of the results from either experiment. However, there is some observed information resulting from a 2006 truck movement study in the county (Rockland County, 2007). In that study, trucks were surveyed at four locations in the county where there are relatively large truck flows. The resulting data provide some coarse indications of internal-internal, external-internal, internal-external and external-external truck movements in the county, at least for movements that pass through the four surveyed locations. These data can be compared to the estimated truck trips in those four categories from both the O-D estimation runs.

In the first experiment, using only link counts constructed from the AADT data, we can compare the assigned volumes using the estimated O-D matrices to the input observations. Figures 5-5 and 5-6 illustrate that comparison for automobiles and trucks, respectively. In general, the estimated O-D matrices, when assigned to the network, would reproduce the observed link counts quite well. There are four links (circled in red in Figure 5-5) where there are observed auto volumes in the 1000-1600 veh/hr range, but no assigned volumes. The location of these links is shown in Figure 5-7 (also circled in red). This is a section of NYS Route 303, north of NYS Route 59. The likely reason for having no assigned auto volumes on these links is that there are no zone centroids in this area. The observed traffic along Route 303 is likely to be mostly originating and terminating in the residential areas along this route, but our representation of the network does not include any centroids for originating or terminating trips in that vicinity. This points to an issue in the choice of number of zones and locations for centroids, but not necessarily a problem with the O-D estimation process.

There is one substantial link volume error in the truck volumes in Figure 5-6. This is a link representing the exit ramp from I-87 Southbound (the NYS Thruway) to I-287 Southbound. There is a complicated merge of traffic from both I-87 and NYS Route 17 into a single entrance to I-287 at this location, which we have represented as separate links, but it is likely that the count reported includes the merged traffic from both, accounting for the larger reported volume.

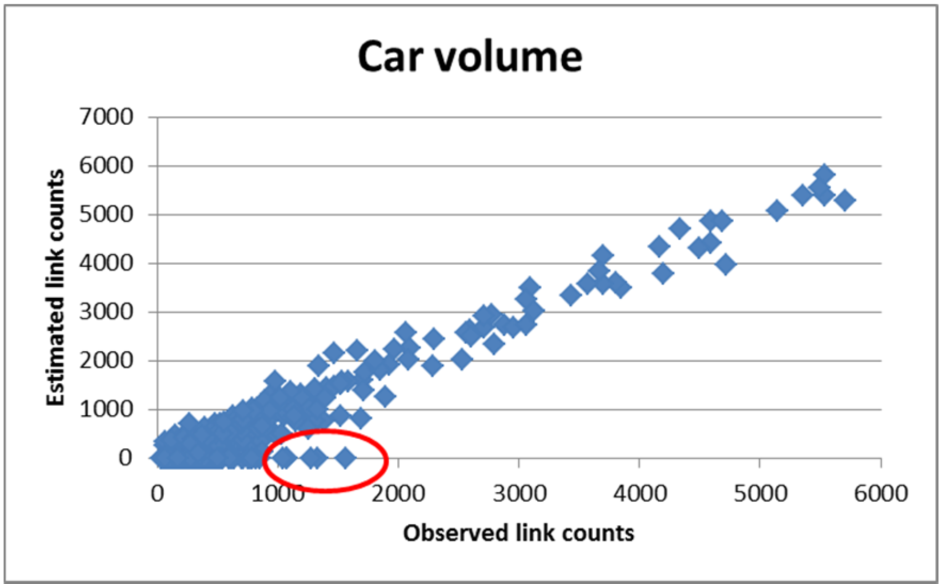

Figure 5-5. Comparison of observed and assigned link counts for automobiles.

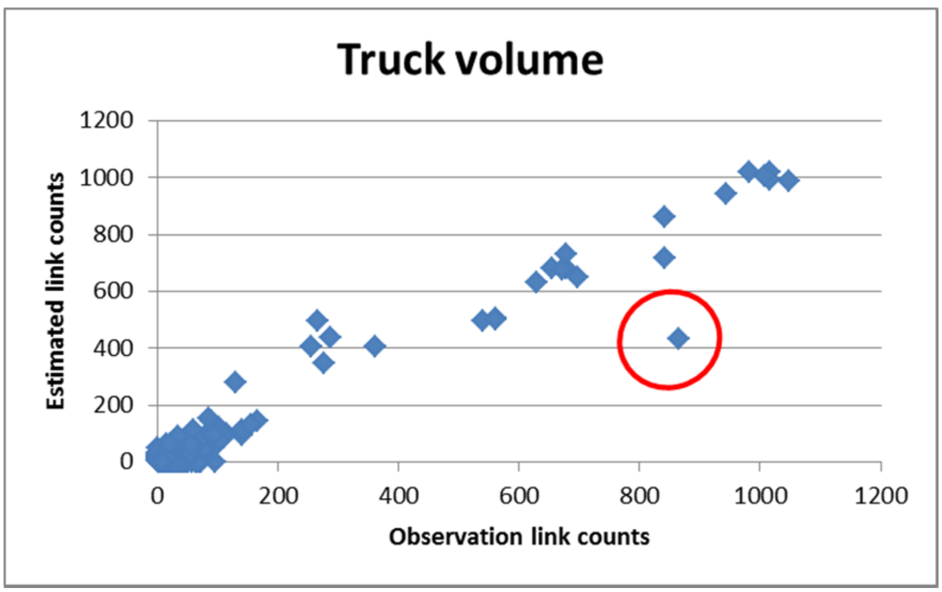

Figure 5-6. Comparison of observed and assigned link counts for trucks.

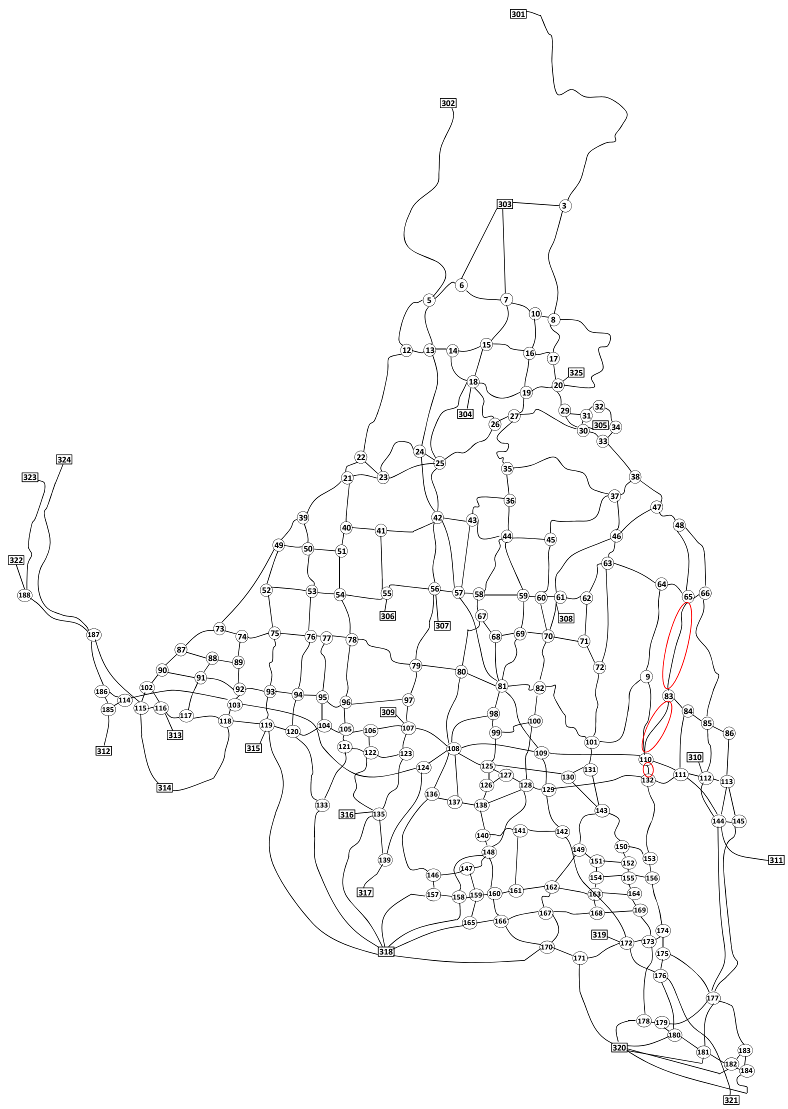

Figure 5-7. Location of links with substantial auto volume errors.

In the 2006 truck movement study in Rockland County, four locations were selected along state or county highways and truck drivers were asked to provide information regarding their current trip. The locations (shown as red circles in Figure 5-8) were selected to focus on truck trips that had at least one end within the county. Many of the external-external truck trips that cross the county are on I-87 or Route 9, and these were not captured in the survey. Thus, the proportion of truck trips from the survey results that are reported as external-external is severely underestimated relative to the total truck flows, but the survey does provide some useful information on the locally-oriented truck flows.

The origin and destination zones in the network can be divided into 14 internal zones within the county and 11 external zones representing entrance and exit points to the network from outside areas. The entries in the truck trip table can be separated into the four groups listed in Table 5-9, and the proportion of total truck trips in each group can be determined. Table 5-9 compares these proportions for the estimated truck O-D table with the proportions estimated from the 2006 survey of truck movements in the county. The primary difference is that the estimated O-D table has many more external-external trips and fewer internal-internal trips than were identified in the survey. Much of this difference may be due to the bias toward excluding external-external trips in the survey, but there may also be a bias toward over-estimating external-external trips in the O-D estimation because the largest link counts are on the Thruway (I-87), which serves primarily external-external trips.

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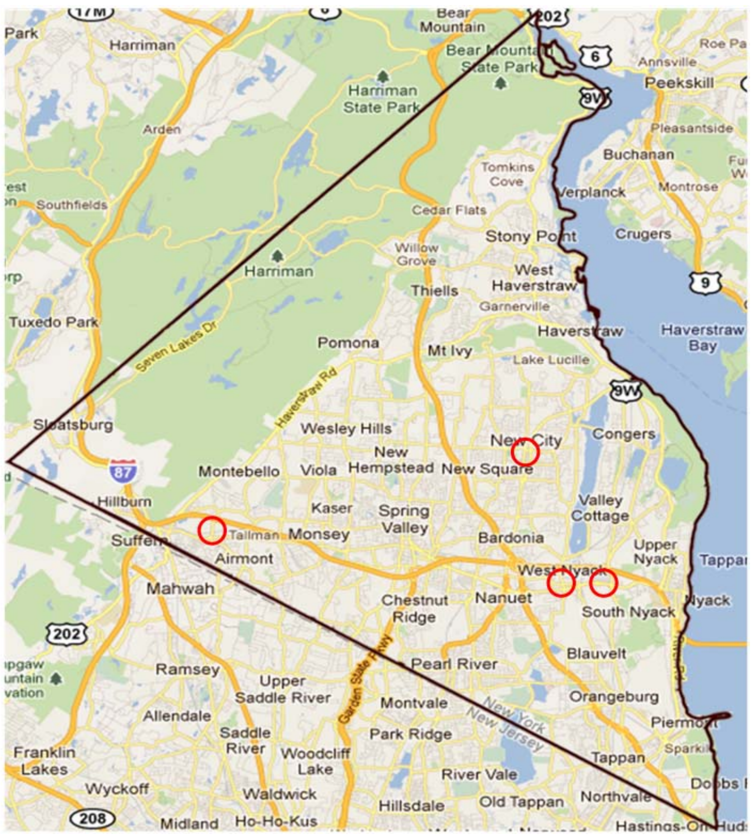

Figure 5-8. Locations of 2006 truck survey stations.

Table 5-9. Estimates of classified truck trip ends based on link counts.

	Proportions of Truck Trips	
Grouping	**O-D Table**	**2006 Survey**
Internal - Internal	0.25	0.48
Internal - External	0.27	0.2
External - Internal	0.2	0.25
External - External	0.28	0.07

The second computational experiment augments the link counts by adding truck trip origins and destinations at the internal zones, using the socio-economic data for trip end estimation. Addition of truck trip-end estimates has almost no effect on the estimated automobile O-D table, as would be expected. However, it results in a significant increase in total truck trips (approximately 32%), with all of the increase focused on trips that have at least one end within the county. Table 5-10 summarizes the breakdown of truck trips after the addition of the trip-end data in a form comparable to Table 5-9.

Table 5-10. Estimates of classified truck trip ends with economic activity data added.

	Proportions of Truck Trips	
Grouping	**O-D Table**	**2006 Survey**
Internal - Internal	0.33	0.48
Internal - External	0.24	0.2
External - Internal	0.22	0.25
External - External	0.21	0.07

The principal change is that there is a much larger proportion of internal-internal trips in the estimated truck O-D table, and a smaller proportion of external-external trips. The effect of the trip-end data is to “fill in” internal-internal truck trips that might not be observed in the link count data that focuses on state and interstate highways. Thus, this data has significant value in improving the overall estimate of the truck O-D table. The total proportion of internal-internal trips remains significantly below that from the 2006 survey, but that is most likely attributable to the bias inherent in the survey data.

5.5 Conclusions

The Rockland County test case demonstrates that the multiclass O-D estimation method developed in this research can successfully estimate trip tables in realistic networks, and thus has significant practical value. It also demonstrates that the addition of truck trip-end estimates based on economic activity data can change the estimated truck table significantly, and thus this data has considerable value.

In the experiments done here, the data from the 2006 truck movement survey in Rockland County has only been used as the basis for evaluation of the overall character of the estimated truck trip table. However, the data represented in that survey is potentially useful in the O-D estimation process itself, even though it has substantial biases toward identifying only internally-based trips. The survey asked truck drivers for their previous stop and next stop, representing the origin and destination of their current trip. These trips were intercepted at specific locations (links). Using the process described in Chapter 3, these observations can be converted to values useful in the O-D estimation by relating them to the probabilities of truck trips (in various size classes) for a given O-D pair appearing on a specific link. In the current case study, only total truck volumes are used, but the survey also collected information on truck class, so that information could also be used in a situation where multiple truck classes are being estimated.

Chapter 6

CONCLUSIONS

Estimating O-D tables for trucks is of substantial interest to transportation planners because there is growing interest in planning for, and managing, truck movements in urban areas. Trucks impose different levels of pavement damage than cars, they have different emission characteristics, different accident patterns, and may be the subject of different types of flow controls. It is also important to recognize that there are several different size classes of trucks, whose O-D patterns are likely to be different from one another. There have been previous efforts to estimate truck O-D matrices independently from automobiles, but the obvious interactions between auto traffic and truck traffic imply that there is a clear need for O-D estimation methods that treat multiple vehicle classes simultaneously.

The O-D estimation method developed in this research is designed to include multiple vehicle classes whose O-D patterns are different, and to accommodate a wider variety of data types than most previous O-D estimation methods, which rely only on link volume counts. For multi-class O-D estimation, it is vital to have data that distinguishes among vehicle classes. Modern traffic sensing technology is providing increasing ability to classify vehicles as they are counted, as well as to create data that are more informative than simple link counts. The O-D estimation method developed here creates an important new tool for transportation planners interested in truck movements (for several size classes) as well as automobile movements, and to take advantage of the increasing sophistication of traffic sensing technology.

In addition to estimating O-D tables for multiple vehicle classes simultaneously, the method developed here also differs in several important ways from previous efforts. First, it does not assume the existence of a “target” trip table to be updated. This is particularly important for truck O-D estimates because existing truck data is often quite sparse and no previous O-D tables may have been estimated. The model developed here does not assume existence of any previous O-D table, although such prior estimates can be inserted if available.

A second important difference in this newly developed method is the use of probit-based stochastic user equilibrium (SUE) assignment as the basis for relating the O-D tables and the resulting link flows. Previous efforts have either assumed a proportional assignment mechanism (especially for trucks), or have been based on deterministic user equilibrium (DUE). The SUE assumption allows for errors in perception on the part of drivers as they make route choice decisions, and thus offers a richer view of equilibrium flows in the network. The underlying assumption of the probit model (that uncertain route characteristics, or driver perceptions of those characteristics, are approximately Normal) is intuitively appealing, but the computations required for assigning traffic to a network using probit-based methods are significantly more complex than the computations for logit-based assignment. This has inhibited more widespread use of the probit model for SUE predictions. However, we have successfully implemented a probit-based computational procedure in the current work.

Use of the SUE assumption for network assignment implies that path probabilities for each O-D pair and vehicle class can be computed. In this way, the SUE assumption is very different from the DUE assumption, where path flows are not uniquely determined. This property creates a mechanism for using a wider variety of observed data types that may be related to path flows, rather than just link counts. This is an important new capability to take advantage of increasing sophistication in traffic sensing technology.

Testing of the method on a widely used small test network has verified the ability of the method developed here to successfully estimate O-D tables when sufficient data are available, and also has lead to important conclusions about the likely success of multi-class O-D estimation under more typical scenarios of data availability. These conclusions are:

1) Without classified link counts, the estimation of truck O-D volumes is completely ineffective. Even if all sensors can differentiate between automobile and trucks (i.e. dual loop counters), the estimates of truck O-D volumes are nearly all incorrect. It is preferable to have 50% of links covered with classification counts than to have only dual loop counts on all links. Using dual loop detectors to substitute for classification counts in estimating O-D tables with more than one truck class is not likely to prove useful.
2) When only single loop detectors exist, even with 100% link coverage, none of the multiclass O-D entries (automobile or truck) is estimated usefully. This is important because single loop counters are currently the dominant source of traffic counts in most urban areas. Moreover, the link coverage in practice is much less than 100%, and under these conditions, it is likely to be a futile exercise to attempt estimation of multiclass O-D tables.
3) Adding turning movement data to link counts is an effective way of increasing the quality of O-D estimates. When only total vehicle turning movements are available, the primary effect is on estimates of automobile (class 1) O-D volumes. If classification counts are available on links, addition of total turning movements at intersections improves the automobile estimation accuracy, although it provides little benefit for the truck classes.
4) When classification information is not available in the link count data, classified turning movements can improve estimation of truck O-D tables dramatically.

5) If vehicle classification counts are available from the link data, classified turning movements can be substituted by cheaper dual turning movements and have similar overall O-D estimation accuracy.
6) There is marked deterioration in the quality of the estimated O-D tables when the observed data contains errors of the magnitude that might normally be expected in practice. This is not unexpected, but this emphasizes two important aspects of the multiclass O-D estimation problem. First, it is important to reduce the errors in the observed data whenever possible. This may mean using better counting technology, or doing more filtering of the data before it is used. Second, in the presence of errors it is necessary to have far more observations to support O-D estimation than would be necessary in an error-free environment. This has implications for sensor location, which includes consideration of both how many sensors, and where they should be located.

A case study of using the method in a realistic setting is performed using Rockland County, New York, as the test network. The principal sources of data for this test case are average daily vehicle counts collected by the New York State Department of Transportation, and estimates of truck trip generation based on land use. The experiments with this test network are designed to answer two basic questions:

1) Does the methodology developed in this project scale reasonably to a much larger network with a much more complex structure and variety of facilities?
2) If basic count data is augmented by commonly available planning data (e.g., land use data) and generic truck trip generation estimates based on that data, how are the estimated O-D tables affected?

The Rockland County test case demonstrates that the multiclass O-D estimation method developed in this research can successfully estimate trip tables in realistic networks, and thus has significant practical value. It also demonstrates that the addition of truck trip-end estimates based on economic activity data can change the estimated truck table significantly, and thus this data has considerable value as an augmentation to observed traffic data.

Several avenues for further research are present. Two general directions of particular interest are sensor location and extension to dynamic O-D table estimation. Because local agencies are unlikely to have instrumentation deployed to observe flow volumes by vehicle class on all network links, resource allocation is an important question. That is, with a limited budget for sensor acquisition and deployment, what locations and types of sensors should be selected to maximize the effectiveness of the data for O-D flow estimation across multiple vehicle classes? This sensor location problem is built upon the O-D estimation process developed here by adding consideration of the information content of different types of observations.

Extension of the methodology to allow dynamic O-D tables is also an important direction. It is clear that both auto flows and truck flows change during the day, and they may change fairly rapidly during some time periods. More effective flow control and traffic management across the day, and across different vehicle classes, requires better understanding of the temporal variation of O-D flows. Much progress has been made in recent years in developing dynamic traffic assignment methods for predicting network flows if the O-D flow rates are known. The challenge is to build methods for inferring the O-D flows from these methods and corresponding real-time observations of traffic flows. There are methods focused on auto traffic only, but little has been done yet in the domain of multiclass dynamic O-D estimation that might apply to both autos and trucks.

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Funded by the U.S. Department of Transportation

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