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September 11th Memorial Program for Regional Transportation Planning

Identification and Modeling of Next Generation Traveler Guidance Systems

Final Report

by

Gitakrishnan Ramadurai
Rensselaer Polytechnic Institute

Academic Advisor: Satish Ukkusuri
Professional Advisor: Todd Westhuis

May 29, 2009

Summary

The objectives of the study were a) to identify the characteristics and architecture of next generation traveler guidance systems, and b) to develop modeling and algorithmic techniques to operationalize the identified architecture.

The research from the study provided preliminary results which developed into three independent research papers each addressing an objective mentioned above. This final report is a compilation of the three independent papers.

The first paper titled "Competitive Service Based 3-tier Architecture for Prescriptive Traveler Guidance System" presents a potential architecture and characteristics of a next generation traveler guidance system. The paper proposes a competitive service based 3-tier architecture for prescriptive traveler guidance system. The proposed TGS will have the following characteristics:

1. Prescriptive guidance based systems; not descriptive information based.
2. Built as a distributed architecture and operated in a competitive market.
3. Enable multi-dimensional choice optimization accounting for heterogeneity and personal preferences.
4. Provide ideal system state based user-optimal guidance that is both fair and efficient.

Substantive arguments are provided and each of the characteristics is described in detail. The presented perspective could potentially translate the promise of personalized, real-time guidance systems to a widely used consumer service.

The second paper titled "Dynamic User Equilibrium Model for Combined Activity-Travel Choices Using Activity-Travel Supernetwork Representation" presents a convenient representation framework for multi-dimensional choice optimization – an important characteristic of the next generation traveler guidance system described in paper 1. In this paper, activity location, time of participation, duration, and route choice decisions are jointly modeled in a single unified dynamic framework referred to as Activity-Travel Networks (ATNs). ATNs is a type of Supernetwork where virtual links representing activity choices are added to augment the travel network to represent additional choice dimensions. Each route in the augmented network represents a set of travel and activity arcs. Therefore, choosing a route is analogous to choosing an activity location, duration, time of participation, and travel route.

The third paper titled "B-Dynamic: An Efficient Algorithm for Dynamic User Equilibrium Assignment in Activity-Travel Networks" presents an efficient algorithm for solving the multi-dimensional choice optimization problem presented in paper 2. A major hurdle for extending the Supernetwork concept to dynamic networks considering activities is that the resulting multi-dimensional dynamic choice problem leads to combinatorially increasing choice dimensions. Therefore existing algorithms that depend on path enumeration such as route-swapping algorithm are difficult to implement even for moderately sized networks. In this paper an alternative algorithm that does not require

path enumeration is presented. The algorithm is a novel extension of Algorithm B (Dial 2006) to dynamic networks and hence referred to as Algorithm B-Dynamic.

The work presented here provide promising yet only initial results towards operationalizing a next generation of traveler guidance systems. Several important issues – both practical as well as technical – need to be solved before the proposed system can be implemented. For example, details on data collection, dissemination, and user interface are very important practical issues that need to be addressed. Further, the efficiency of the proposed modeling and algorithm framework has to be improved by several orders of magnitude for implementing them to provide real-time guidance.

I take this opportunity to thank my academic advisor, Dr. Satish Ukkusuri and my professional advisor, Todd Westhuis for their guidance and support throughout the study. I also thank UTRC and NYMTC for providing this opportunity to work on an exciting research topic.

Competitive Service Based 3-tier Architecture for Prescriptive Traveler Guidance System

Gitakrishnan Ramadurai
PhD Student
4002 Jonsson Engineering Center
Rensselaer Polytechnic Institute
Troy, NY 12180
ramadg@rpi.edu
(Corresponding Author)

Satish V Ukkusuri, Ph.D.
Assistant Professor
Blitman Career Development Chair Professor
4032 Jonsson Engineering Center
Rensselaer Polytechnic Institute
Troy, NY 12180
Phone : (518) 276 6033
Fax : (518) 276 4833
Email : ukkuss@rpi.edu

Word Count: 7143 Words + 1 Table + 4 Figures

ABSTRACT:

The paper discusses competitive service based 3-tier architecture for prescriptive traveler guidance system. The proposed TGS will have the following characteristics:

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We provide substantive arguments and describe each of the characteristic in detail. The presented perspective could potentially translate the promise of personalized, real-time guidance systems to a widely used consumer service.

1.0 INTRODUCTION

The rapid advances in communication and computing technology have enabled several Intelligent Transportation System applications. Foremost of these systems is Advanced Traveler Information Systems (ATIS) and the related Route Guidance Systems (RGS). Traveler information systems were envisaged as a tool that will assist travelers make efficient travel decision by providing network state information both pre-trip and en-route. Such information could be both descriptive (such as describing congestion levels on the network) as well as prescriptive (such as informing drivers to avoid certain roads due to incidents). Though several studies have examined different components of ATIS over the past twenty years (see *(1)* for a comprehensive review), recent developments in mobile and handheld location devices combined with ubiquitous wireless connectivity have opened up opportunities towards developing the next generation of ATIS. We call this next generation ATIS system as Traveler Guidance Systems. The term traveler guidance system is adopted since it is envisaged that the role of these systems in future will be broader than route guidance alone.

This paper discusses the characteristics and architecture of the proposed next generation traveler guidance systems (TGS). We present a perspective that could potentially translate the promise of personalized, real-time guidance systems to a widely used consumer service. We define the characteristics and provide substantive arguments to support them. While we do not claim that the presented perspective is a unique method to success or that the perspective is comprehensive, we do believe the characteristics identified will form the basis for a successful implementation of next generation of navigation systems. Also, while each of the individual characteristics that have been identified may not be a new contribution, this is the first attempt that integrates the different characteristics to ensure an overall framework that is likely to succeed.

The proposed TGS will have the following characteristics:

1. Prescriptive guidance based systems; not descriptive information based.
2. Built as a distributed architecture and operated in a competitive market.
3. Enable multi-dimensional choice optimization accounting for heterogeneity and personal preferences.

4. Provide ideal system state based user-optimal guidance that is both fair and efficient.

The paper is organized thus: first we provide a background on navigation and traveler information systems. We provide a brief but rigorous overview of important past research efforts and draw lessons from their failures to support our perspective. We then discuss each of the above characteristics in detail and provide substantive arguments to support them. The final section summarizes the paper.

2.0 BACKGROUND

In-vehicle navigation system has remained the most promising technology to tackle urban congestion for over forty years. According to Boyce (*2*), the first concerted research effort to develop a route guidance system was the Electronic Route Guidance System (ERGS) by the Bureau of Public Roads of the US Department of Transportation in 1967. It is interesting to note that this early system was referred to as a 'guidance' system instead of an information system. As such, guidance and information are fundamentally different methods of communication; guidance is ***prescriptive*** while information is ***descriptive***. Consequently, the paradigm that emerges based on whether a system is for guidance or for information delivery will significantly impact the architecture as well as the models that define the system. We discuss this important classification in detail in Section 3.1.

The architects of ERGS realized the importance of providing guidance accounting for the changes in traffic flow over time – referred to as "dynamic" route guidance problem. The ERGS was not successful primarily because efficient algorithms to estimate minimum paths in large scale networks accounting for the dynamic changes in traffic were unavailable. Boyce (*2*) describes other subsequent research efforts in Japan and Germany – these projects too were not finally implemented because of their limited capabilities (static guidance in Japan) or high costs of installing roadway devices (in Germany). Boyce (*2*) also reviews the first commercially available navigation device known as Etak navigator. The system was primitive since it provided only static map tracking (no guidance) and expensive (approximately $2000). The system may be viewed as the predecessor to current static GPS-based navigation devices that are available in the market. The period from 1967-1987 may be viewed as the first-phase of vehicle navigation system research. The initial promise of the technology was met with severe limitations both from technological as well as analytical modeling perspectives.

Boyce (*2*) highlights necessary technology to take the navigational aids developed in the first-phase described above into an advanced route guidance system. He describes three major requirements: i) a two-way communication system, ii) comprehensive traffic monitoring system, and iii) central analysis unit comprising supercomputers to compute best paths. Further, he identifies the need to address technical questions such as whether the existing static network equilibrium techniques can be extended to a dynamic formulation and whether the traffic monitoring system will provide sufficiently accurate estimates of route travel times. Addressing these challenges were the primary focus of research efforts in the next phase of research in vehicle navigation systems.

The second-phase of vehicle navigation system research may be classified from the late 80s to the current period. This second-phase saw several important paradigm shifts due to many factors. First, improvements to traffic monitoring and communication technologies enabled real-time collection and dissemination of traffic data. Second, public agencies and the government pioneered the development of Intelligent Transportation System technology and supported several research projects. Third, research on the analytical and algorithmic modeling of transportation networks accounting for “dynamics” made rapid strides. This second phase also saw the emergence of alternative business models for traveler information dissemination. Accordingly, to encompass this broader area of research into information dissemination to assist travelers, a new term referred to as advanced traveler information systems (ATIS) was coined. In addition to the private in-vehicle navigation device, alternative communication mediums such as public information through the radio and TV, telephone, Internet, and variable message signs (VMS) emerged. A FHWA report (*3*; table 4-1) provides a summary of ATIS, the degree of deployment, limitations, and future promise. Interestingly alternative communication mediums such as internet, television and dynamic message signs were considered to be successful implementation of ATIS while in-vehicle navigation systems were considered to “hold promise”. At the turn of the millennium, the transition from a focus on in-vehicle guidance in the 70’s to public dissemination of ‘descriptive’ information was complete. This transition also resulted in a shift in business model. The responsibility for maintaining ATIS services shifted from private industries to public agencies. This shift is acknowledged by Sussman (*4*) in his retrospective on the 1992 ITS strategic plan. The 1992 strategic plan predicted private-sector organizations providing traveler information to individual users as a profit-making activity. He declares (*4*) that “it is clear that making a profit in this business is very hard”. He further states that it could be because the public do not value more sophisticated ATIS information than what is commonly available through radio.

Intuitively it can be argued that the emergence of public dissemination of traveler information delivered only part of the promise originally envisaged by in-vehicle guidance systems. All the alternative communication mediums are restricted spatially, temporally, and have put the onus on the traveler to seek information. The any-time, any-where guidance of in-vehicle navigation devices is not possible through these alternative communication mediums. In order for the true potential of in-vehicle navigation to be realized there is a need for the public to realize the additional value in guidance systems besides information from alternative mediums. We anticipate that this change will trigger the third phase of research into navigation systems.

Over the last few years, there has been an important change in market trends for personal/vehicle navigation devices. ABI research (*5*), a market research firm, predicts a potential growth in GPS navigation devices from 140 million devices in 2007 to over 600 million by 2012. Similarly mobile devices with integrated GPS are likely to grow from 180 million units in 2007 to 720 million units by 2011 according to In-Stat (*6*), another market research firm. In a recent survey of 22 experts (*7*), GPS and Personal Travel Assistant were ranked as the highest among five emerging transportation technologies for congestion reduction in the next twenty years. The average predicted market penetration for GPS and personal travel assistants was over 50% after 10 years, and over 80% after 20 years (*7*).

The coming decade could form a new phase in the research into ATIS. The promise of in-vehicle navigation systems may come to be realized to its full potential.

The primary technology drivers such as device technology as well as communication architecture are significantly advanced to support this paradigm shift. The critical factors that need to be addressed are the development of suitable business models and analytical methods to determine efficient and fair guidance.

In this paper, we present a next generation ATIS system referred to as Traveler Guidance Systems. The term traveler guidance system is adopted since it is envisaged that the role of these systems in future will be broader than route guidance alone.

3.0 TRAVELER GUIDANCE SYSTEMS: CHARACTERISTICS

The US DOT has identified several future goals for information services as part of ITS initiatives (*8, 9*). The TIS in the future should provide end-to-end *personalized* transportation trip information including time, cost, weather, environmental impacts and support personal lifestyle choices. It should be ubiquitous for all modes and available in different formats. From the network manager viewpoint, the future TIS should assist in optimal operational performance (including the use of pricing).

Given the above different objectives for the future traveler information systems, we propose a traveler guidance system that will address the following objectives:

- Provision of any-time, any-where, any-mode services
- Account for individual preferences including multi-criterion objectives
- Share the burden of infrastructure investment with the private SP and the consumers
- Prescriptive guidance accounting for individual needs.
- A more direct control for the network manager in terms of controlling supply-side parameters.

Below, we provide details about each of the characteristics of the TGS listed in Section 1.

3.1 Prescriptive Guidance

ATIS can broadly be divided into two types: descriptive and prescriptive. Descriptive ATIS pertains to providing information about delays, incidents, and congestion maps and allowing the traveler to make a more informed decision. Prescriptive guidance on the other hand makes the decision for the traveler and informs her of the “optimal” choice. The proposed TGS will be based on prescriptive guidance with descriptive information being provided when the traveler requests additional information supporting the prescriptive guidance. We argue the advantages of prescriptive guidance over descriptive information below.

Current state-of-the-art of ATIS can be termed as descriptive. However several studies have questioned the true benefit of providing descriptive information to travelers. Arnott et al. (*10*) address the fundamental question of whether information provision reduces traffic congestion. In a simplified setting, they demonstrate that when all individuals are provided the same information, everyone is worse off. Ben-akiva et al. (*11*) discuss three behavioral phenomena that may negate beneficial effects of improved information. These are oversaturation, overreaction, and concentration. Oversaturation refers to information overload under which individuals are unable to make optimal decisions. Travel choice process is a complex game theoretic problem with multiple decision dimensions and complex interactions among several thousand decision makers. Solving the dynamic traffic assignment problem that determines optimal route and departure time for a given origin-destination demand requires complex simulation based algorithms that require several iterations to obtain reasonable solutions; human computation, which is limited, will therefore likely be less efficient. In fact even after accounting for day-to-day learning capabilities of travelers, oversaturation is more likely the norm rather than exception due to the dynamic and stochastic nature of transportation networks. Overreaction refers to most travelers deciding to switch uniformly and transferring congestion from one road or time period to another. Concentration occurs when several travelers converge to the 'best' path thereby congesting it. Mahmassani and Herman (*12*) have shown there is an optimal fraction of individuals who should receive information. Koutsopoulos and Xu (*13*) report that as the percentage of informed vehicles approach 100%, the travel times are actually higher than the base case of not providing any information. Clearly providing current network information is not likely to help all individuals improve their travel times in all cases.

An alternative to providing current travel time information is estimating future travel times based on recent history (*14*, *15*, *16*). However Kitamura and Nakayama (*17*) argue that such an approach does not consider the cognitive process through which drivers make travel time predictions. They draw a parallel between the route choice problem and the minority game. They demonstrate through examples that "predictive travel time information provided to drivers to assist their route choices may not influence network flow; under the condition that route choices are made repeatedly by the same group of drivers, the drivers self-organize to reach the same network equilibrium irrespective of the contents of predictive information given."

Ben-akiva et al. (*11*) note that providing predictive information where models anticipate likely individual reactions to information provided are likely to decrease the possibility of overreaction. However the problem of concentration still persists and may be solved only by providing direct routing information (*11*). The other solution suggested by the authors is obtaining information from drivers directly regarding their intentions. It is not clear whether predictive information will lead to consistent predictions – that is individuals could modify their decisions based on the predictions thus invalidating the original prediction. Few studies (*18, 19, 20*) have examined consistent "guidance" systems that account for individual behavioral response. However, there are no field trials available to prove their validity. In fact, laboratory simulation based experiments that have examined the effect of information on driver behavior has provided results to the contrary. Ramadurai and Ukkusuri (*21*) conducted an online game where players were required to choose their departure time in real-time. The games were repeated with and without information about network state. When network state information was provided, overreaction as well as overall reduced payoff were observed. In summary, descriptive information provision – be it real-time, estimated, or predictive – is not likely to provide substantial improvements to travelers as the market penetration improves to higher levels. These problems are considerably reduced, if not eliminated, when travelers are provided with prescriptive guidance. In addition, prescriptive guidance has several other desirable features as described below.

Prescriptive guidance is less distracting compared to descriptive information. Based on driving simulator studies (*22*) it was found that drivers prefer short, simple information. Also, as we move towards the age of ubiquitous connectivity, activities will become even more fragmented and travel plans increasingly ad-hoc. Optimization of travel choices may have to be performed in real-time. Travelers will turn towards computers to perform optimal travel choices and expect to obtain prescriptive guidance as a result. Other potential future scenarios such as dynamic pricing, multi-criterion objective including environmentally conscious decision making also require substantial computational and knowledge that will require computer algorithms to provide travelers guidance. Prescriptive guidance also has a greater likelihood of eliminating idiosyncratic decisions such as freeway bias where travelers often choose freeway routes as opposed to shorter non-freeway routes (*23*).

A primary criticism of prescriptive guidance is non-compliance – that is travelers are not likely to follow the guidance. This may arise because travelers have certain inherent preferences or because they do not trust the information provided. Each of these criticisms is addressed in the proposed system. Since guidance is provided at an individual level, one can obtain the intrinsic individual preferences and incorporate these preferences in determining guidance. The lack of trust may be addressed by providing travelers with additional descriptive information. In fact, combining prescriptive recommendation with descriptive information justifying the recommendation is likely to result in greater compliance (*24*).

Further, there have been several survey-based studies (*25*; *26*; *27; 28*) on the acceptance of prescriptive guidance compared to descriptive information. Khattak et al. (*28*) report that, when respondents were presented with a scenario where pre-trip information is provided, though greater change (in terms of departure time, route, or mode) occurs when quantitative information and predictive information are provided, significant change in travel patterns (route or mode change as opposed to departing earlier or later) occur only when prescriptive information is provided. Similarly Polydoropoulou (*27*) report that when individuals were presented with a hypothetical scenario where they are provided delay information through in-vehicle devices en-route, "twenty-seven percent of travelers would switch to the alternative route when qualitative information is provided to them. This increases to 52 percent when quantitative information for the usual route is provided, 55 percent when predictive information for the usual route is provided, 58 percent when delay information on the usual route and travel time on the best alternative route are available, and 61 percent when prescriptive information to take the alternative route is provided." Mehndiratta et al. (*25*) report that "at least in a familiar commuting context, users are split in their opinions whether they prefer to receive routing advice or simply advice about traffic delays to inform their own routing decisions" and even among respondents who disliked guidance, the reasons were because "the drivers did not trust routing (they thought that they could do better because from their own driving experience they often had alternative routes of their own), and they implied that their personal objective functions did not always coincide with the algorithms used by the routing device." In summary, prescriptive guidance that accounts for personal preferences and provides strong justification using descriptive data for the choices made are more likely to be accepted by travelers than pure descriptive information. The rest of the characteristics of the proposed guidance systems achieve this objective of user optimal guidance accounting for personal preference. These are discussed in detail below.

3.2 Three-tier Distributed Architecture

Existing studies divide the ATIS into two categories: a) centralized architecture with on-board (vehicular) components, road-side units, and centralized control systems, and b) decentralized architecture where all information collection, storage, and communication are between individual vehicles. Decentralized architecture is particularly attractive since it transfers the monetary burden of installation and maintenance of the infrastructure to the users. However, decentralized architecture may be inefficient since each individual may make selfish decisions that could be sub-optimal for self (because of imprecise knowledge) and system (*29, 30*).

In order to obtain a better idea on how the future architecture could emerge it is important to identify the key players in alternative business models. In (*31*), the following alternative business models are presented:

- Public-sector funded
- Franchise operations
- Private-sector operated and funded models
- Value-added reseller models
- Business-to-business models

They (*31*) also report that key shifts in both the public sector arena as well as the private sector marketplace are evident. Public agencies are taking on more responsibility for aggregating data to support systems and are also developing innovative applications to deliver personalized and enhanced information to travelers. For example, NYSDOT as part of the services under the 511 program is communicating information to the traveling public via e-notification services. The private sector which was leading efforts for data fusion and dissemination have increased their activity in terms of data collection, including infrastructure-based, probe vehicle data, and aggregated, multi-source data. Therefore the traditional responsibilities of the different agents are changing. The USDOT ITS vision statement (*32*) provides a possible result of this change.

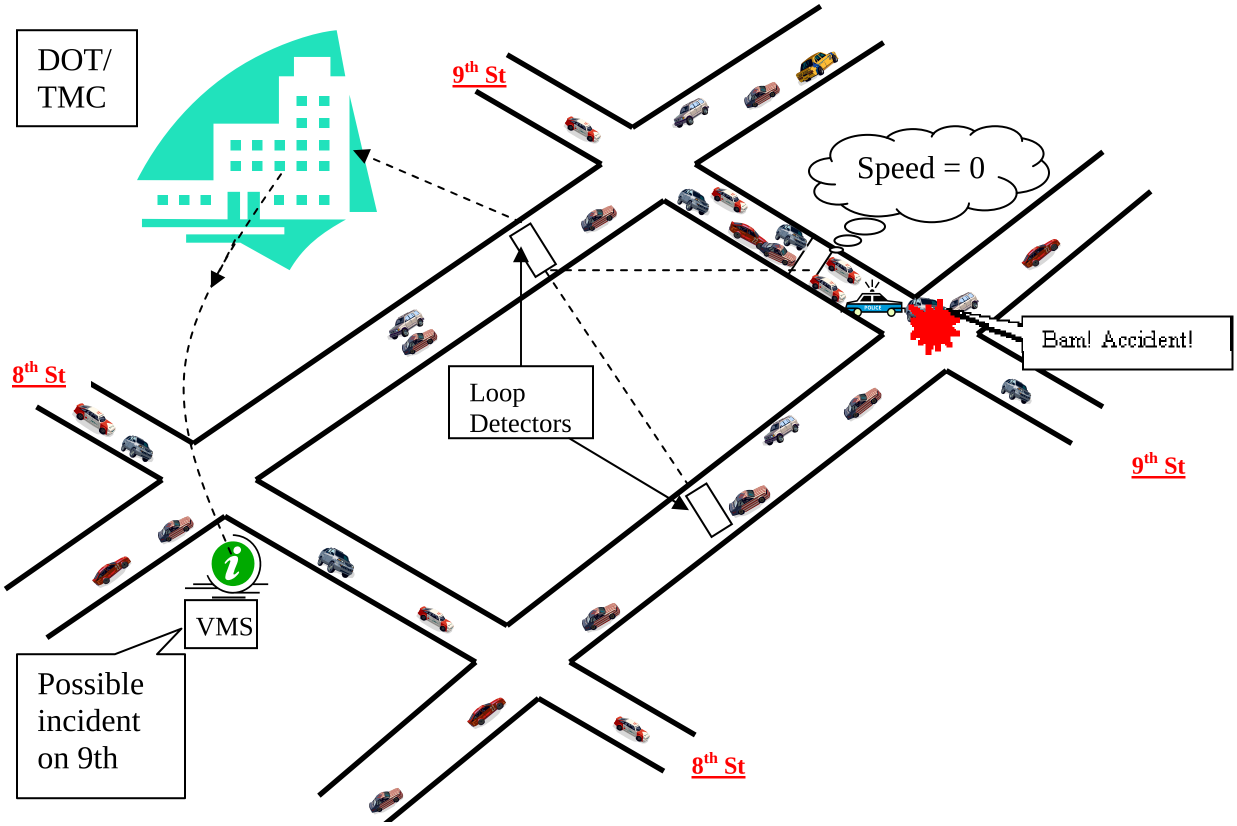

Figure 1: Centralized System

The USDOT ITS vision statement predicts that 15 years in the future: "Commercial entities, in the form of "Information Service Providers", or ISPs, have been built upon the early public sector foundations of ITS. These ISPs provide value-added services, by collecting data from various sources and creating valuable information products and services that consumers now see as just as necessary as their TV, on-line computer, and telephone services."

It increasingly appears that the future of ATIS lies in an architecture arising from a partnership between public transportation agencies and private service providers. Also, shifting the onus of information and guidance dissemination to private sector absolves the public agencies from dealing with the dilemma of equitable information provision. Allowing for a competitive market based operation where different service providers (SPs) compete for market share will enable travelers to get the best service possible; also the competition will force service providers to cater to individual traveler preferences. While public-private sector cooperation is not new in urban transportation (*33*), what will be innovative in the emerging guidance systems is that the *traveler* will be a central player along with the public and private organizations. The distributed three-tier architecture (DOTs – SP – Traveler) and a competitive market operation is the most likely future ATIS strategy that can address the problems of ensuring fair and efficient guidance accounting for personal preferences to all travelers.

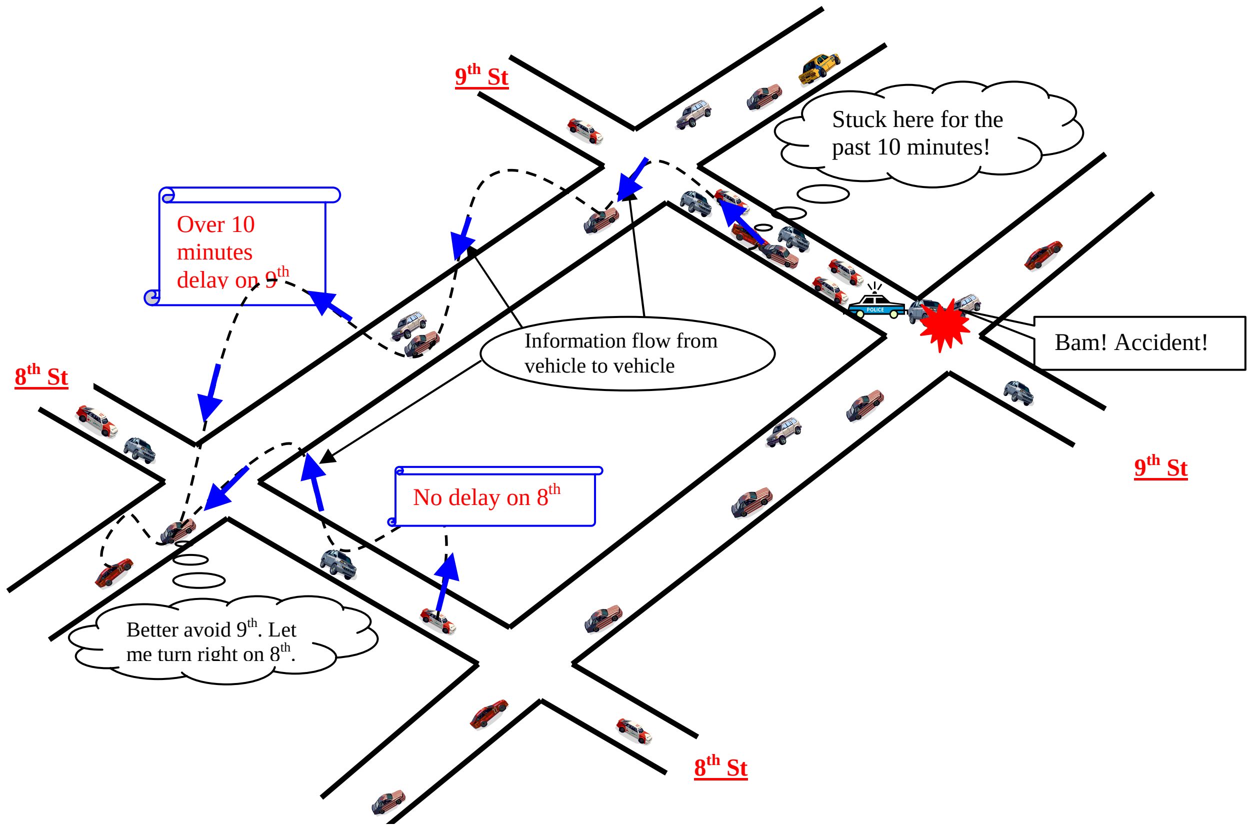

Figure 2: Decentralized System

Traditionally two types of architectures have been proposed for ATIS: centralized and decentralized architectures. The centralized architecture has a central agency – TMCs/ DOTs/ MPOs – collecting data on network state and disseminating messages to the public. All control in terms of data collection and dissemination is at the central location. In decentralized architecture, vehicles and travelers act as both data collection and dissemination agents. Prescriptive guidance is not possible in decentralized architecture since individuals make their decisions based on the information they obtain; there is no central agent providing instructions. Figures 1 and 2 are schematic representation of these two architectures. The scenario depicts an accident occurring on 9th street. In the centralized architecture, the incident is detected by the loop detectors and this information is conveyed to the central traffic controller. The information is subsequently broadcast on variable message signs and individuals make choices based on the information broadcast. In the decentralized architecture, the information is transmitted from vehicle-to-vehicle and the decision is left to the individual.

In this section, we propose a three-tiered distributed architecture for the next generation of TGS. A preliminary layout of such architecture has been reported in the literature earlier (*34*, *35*) and is shown in Figure 3. However Adler et al. (*34*) ignore possible data collection function of individuals/ISPs. While Oh and Jayakrishnan (*35*) recognize the possibility of data collection by individuals, their overall system is still based on descriptive information dissemination. Here, we recognize that the private sector is beginning to also operate probe-vehicle based data collection. And at the same time the SP disseminate prescriptive guidance to individuals. It is anticipated that in the future due to increased penetration of personal location based devices, and due to the rising cost for public agencies to maintain and operate the data collection infrastructure, hybrid architecture where both public and private agencies participate in data collection will emerge.

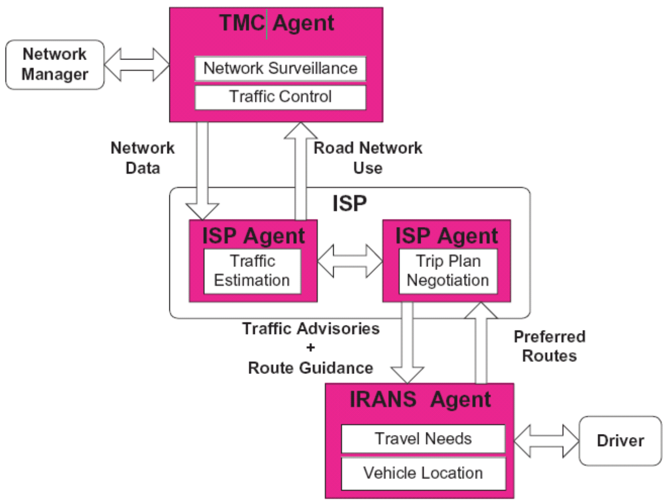

Figure 3. Distributed Architecture in (*34*)

Also, in the distributed architecture, several private agencies (referred to earlier as Service Providers) communicate to individuals who are subscribed to their services. Since the presence of several private service providers (SPs) will spawn a competitive market for value-added services, we hypothesize that they will operate under a competitive economic market framework. The primary driver of the architecture will be the end user – the subscribers to the service. As the subscriber begins to see benefits from the system in terms of saved time and fuel (particularly with increasing gas prices), s/he is likely to pay for the service and at the same time demand quality for the price paid. This also enables the competitive market to operate efficiently.

The distributed architecture has several advantages. It transfers a portion of the cost of sensor and communication infrastructure installation and maintenance to the private service provider and consumer. It allows for more efficient customer service due to the competitive framework. Further, the public agency may choose to operate the network by controlling supply side parameters such that they are able to ensure certain system wide performance targets. Communicating and ensuring compliance of such system wide performance targets is easier when dealing with few SP agents instead of thousands of travelers directly.

Table 1 below summarizes the role of each of the agents in the distributed architecture. The public agencies represented by the DOTs / MPOs / TMCs / Transit authorities collect data from on-road sensors and broadcast the information directly to the SPs. They may also broadcast information through various publicly accessible sources especially for the population who have not subscribed to the paid services from the SPs. The information disseminated by the public agency includes network state information such as delays, congestion, incidents and future supply decisions such as lane closures, ramp metering scheme, and toll pricing. The second-tier, comprising the SPs, act as an intermediary between the public agency and the traveler. The SPs could also collect data either from privately operated companies which have their own road sensors or from probe vehicles that are part of their clientele. The SPs are expected to gauge traveler preferences accurately and deliver personalized guidance information. They are also expected to perform network wide optimization ensuring both travelers as well as public agencies criterions are met. They could also communicate to the public agency either to provide network state data obtained from their data collection infrastructure or negotiate ‘deals’ with regard to future network supply utilization. Finally, the traveler reaps the benefit of the proposed architecture as he obtains personalized, optimal travel guidance. Figure 4 provides a schematic of the distributed architecture.

Table 1. Three-tier Distributed Architecture

Agents	Data Collection	Communication	What Information?
Public agency - DOTs/MPOs	Yes: on-road sensors, including centralized consolidation	Broadcast to SPs; also through VMS, Web, SMS, 511 to public	Disseminate: Network state, future supply decisions
SP	Possibly install on-road sensors, but will definitely include probe vehicle data consolidation	Personalized communication to individuals (clients); possible communication with DOTs/MPOs	To clients: guidance information To DOTs/MPOs: network state from probe data, negotiation for using future network supply
Traveler	Yes - serve as vehicular probes	To ISPs – probe data	To ISPs: current traffic conditions; from ISPs: guidance information

Given the possibility that several individuals may not subscribe to the services in the short-term, it is important for alternative sources of information communication be in place. It is therefore anticipated that a mixture of Centralized and Distributed architectures will emerge in the short-term with a completely distributed architecture emerging later.

A key to the success of the proposed architecture is the question of profitability for the private SP to enter the market. Willingness-to-pay for ATIS services has been an important question that has been studied in the past and the studies report a general willingness to pay for good quality information (*26*, *36*, *37*). It can be safely said that with the recent increasing trends in sale of personal navigation devices as well as mobile phones with GPS, the market will expand further. Already real-time information based guidance is being provided by few private firms in the United States (for example Inrix, Telenav, and Dash). However, they provide reactive descriptive information based on current network state or statistical regression based predictive models that do not account for user behavior. The potential for private SP to enter the prescriptive guidance based TGS market is high. The current architecture and technology support it; however, the modeling and algorithmic component of TGS need to be further developed. This forms the focus of the next two sections.

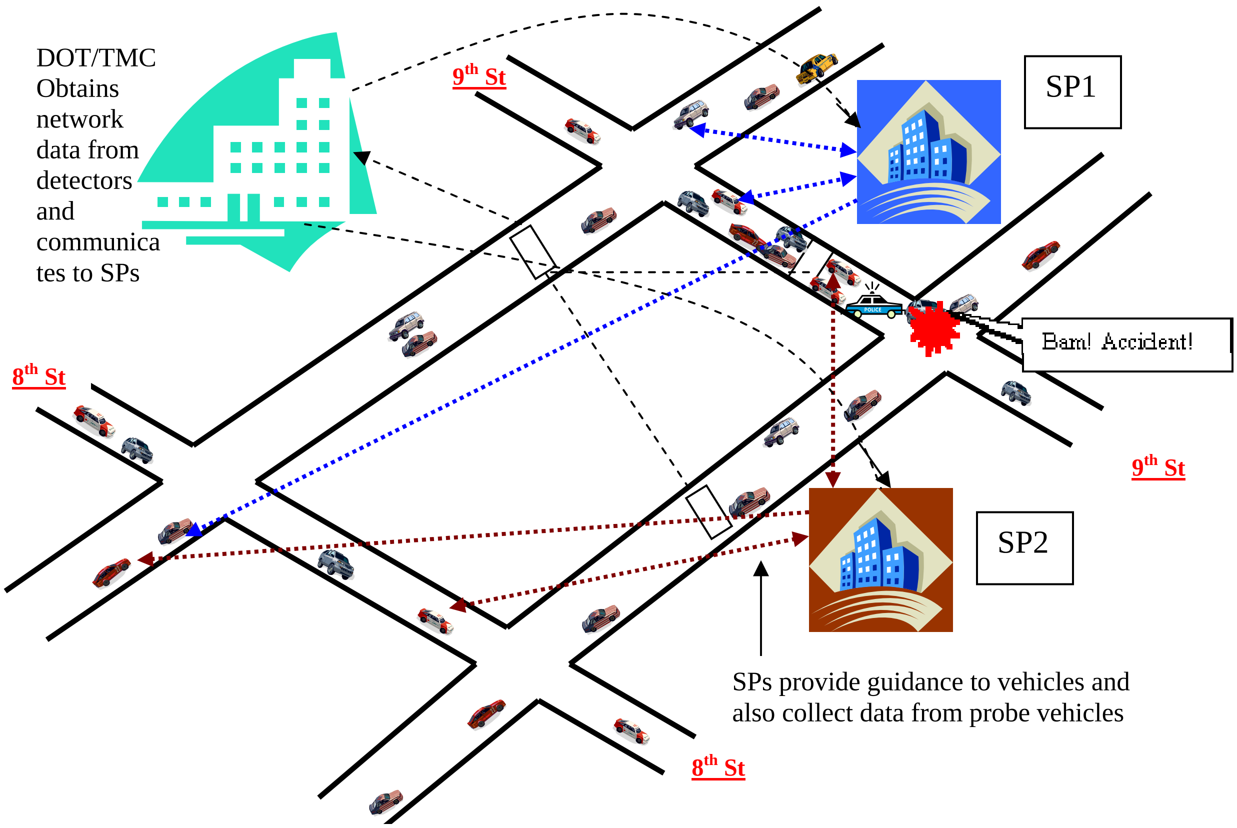

Figure 4: Distributed Architecture

3.4 Multi-dimensional Choice Optimization and Personal Preferences

Travel is a multi-dimensional choice process. Typical travel choice decisions include: whether or not to travel, what activities to participate in, where to engage in the activities (destination), when to depart, how long to engage in each activity, which mode and route to choose. Each of these decisions are influenced among other factors by personal preferences, degree of congestion on the network, comfort and cost of the mode, stochastic disturbances on the network, availability of parking etc.

A severe threat to prescriptive route guidance is when guidance is provided ignoring the other dimensions of the travel process such as activity location, duration, and mode. Current traveler information systems are reactive; they provide route guidance after the driver enters the network thereby losing the opportunity to schedule travel based on network congestion. In other words, existing systems manage congestion; they do not prevent it. The next generation of traveler guidance systems will provide information across, and thereby influence, multiple dimensions of travel choice including departure time, destination, and route choice. It is anticipated that this would significantly improve system utilization and throughput due to the increased flexibility afforded by optimizing over a multidimensional choice space. The travelers develop a trust-based relationship with their SP and provide their activity schedules. The SP in turn collates each client's activity schedule and performs a multi-criterion equilibrium assignment based on generalized utility measures over an extended network framework similar to

Supernetworks (*38, 39*). The process is expected to be dynamic and the traveler can accept, reject, or modify guidance provided by the SP. The SP will also obtain advanced information from the public agencies regarding future network supply decisions and factor them in their calculations.

Travelers are likely to have different personal preferences. For example, in a survey undertaken at the University of Leeds (*40*), it was observed that time minimization was quoted by less than half the respondents as their primary objective. Avoiding congestion and certainty of arrival time were also important. Therefore, for the next generation of TGS to succeed, it must allow travelers to tailor the guidance they obtain based on their personal preferences. Maintaining their client base in a competitive market will spur the SP to provide increased personalization options to travelers. The process can also work in reverse where the SP by providing options such as minimize pollution or save fuel provides the traveler with increased power in making their travel choices.

Further, travelers can define sophisticated optimization objectives such as minimization of weighted function comprising fuel consumption, pollution emission, transit use for at least part of the journey, or potential for car sharing and ride matching among others. In this context the proposed architecture not only allows for multi-dimensional choice optimization but allows multi-criterion objective based optimization. Therefore, the proposed system could potentially enable an informed, welfare-maximization based transportation system.

It is envisaged that the TGS will be a software tool similar to an activity scheduler that is accessible from any device with network connection capabilities. Therefore an individual with a mobile phone or personal navigation device that can connect to the Internet will be able to access, plan, and obtain guidance information any-time, any-where. Such an interface will also provide increased flexibility to the user to enter preferences. Objectives mentioned above as well as constraints to the user may be input; for example, if the traveler prefers to always travel through a particular link he can request the TGS to account for the link in the prescribed route. Obtaining clear and detailed input from the user and providing guidance such that they meet the input requirements as closely as possible and providing descriptive information to support the prescribed decision is likely to improve compliance and build trust with the user. Ben-akiva et al. (*11*) remark that "obtaining information from drivers regarding their intentions may produce better predictions." This is precisely the objective of the proposed TGS.

3.5 Ideal System State based User Optimal Guidance

Finally, it is important to discuss how SPs decide their optimal guidance strategy. Traditional equilibrium based traffic assignment prescribes two major strategies: user equilibrium and system optimal. User equilibrium strategies arise when no individual can unilaterally shift and improve his travel time. This is similar to the Nash Equilibrium concept. On the other hand, system optimal ensures that all individuals as a collective whole, experience the least amount of travel time. System optimal by design can penalize few of the users severely while providing others much better travel times. System optimal guidance is unlikely to be accepted by all travelers and is likely to fail. On the other hand user equilibrium - a result of selfish behavior of each traveler - has been shown to be inefficient for the system as a whole when compared to the system optimum objective. In the proposed TGS individual traveler trusts his SP to provide guidance for him. While the SP cannot provide the traveler with sub-optimal decisions (since a competing SP could steal the client with a better offer), the SP can strive to ensure that each traveler obtains near-optimal guidance but at the same time system wide performance is maintained at acceptable levels. We refer to such solutions as ideal system state based user optimal decisions. We know such states exist in static transportation networks. For example, consider the problem of Braess's paradox (*41*). On the addition of a single link to an existing network, every user observes an increase in travel time if they behave selfishly (consistent with user equilibrium). The reverse problem teaches us that, it is possible for all users to be better off if they cooperatively agree to avoid the malicious link. Similar concept of obtaining improved system objective without adversely affecting all users has been explored in the static context by Jahn et al. (*42*). They motivate a route-guidance system based on a constrained system optimal formulation that maximizes the total travel time subject to certain user constraints. An ideal system state (ISS) is defined as the optimal system state such that no user is worse off compared to his/her user equilibrium state. By definition the ISS ensures fairness to all individuals – all individuals are guaranteed to experience costs that are no worse than the user equilibrium cost. Also, given the constraint that no user is worse off, the ideal system state represents a constrained system optimal state that is favorable from a network manager's view. Within this context it can be argued that the ISS is efficient as compared to the user equilibrium solution.

However, multi-dimensional equilibrium under a multi-criterion objective function is a challenging problem especially under a dynamic (time-dependent) context. Research studies are required to address these questions and will remain an important hurdle that would have to be crossed to ensure operational success for the proposed traveler guidance systems.

4.0 SUMMARY

In this paper we presented a competitive service-based, 3-tier architecture for prescriptive traveler guidance system (TGS). Increasing sales of personal navigation devices have enabled the possibility of personalized, real-time guidance. The true potential of traveler guidance and information systems may be reached by properly utilizing these personal navigation device capabilities.

Here, we presented an architecture and desirable characteristics of one such potential guidance system. The proposed TGS will have the following characteristics:

1. Prescriptive guidance based systems; not descriptive information based.
2. Built as a distributed architecture and operated in a competitive market.
3. Enable multi-dimensional choice optimization accounting for heterogeneity and personal preferences.
4. Provide ideal system state based user-optimal guidance that is both fair and efficient.

Substantive arguments were presented for the necessity of the above characteristics based on past literature and intuition. However, to enable the architecture, several challenges need to be overcome. Foremost of the challenges is the need for more accurate modeling methodologies to account for multi-dimension, multi-criteria objectives and ideal system state based solutions. A second challenge is scalability. A scalable prototype system has to be demonstrated in reasonably sized transportation networks. Addressing these challenges should be the next steps to transform the proposed traveler guidance system to a real service for the end-user.

Acknowledgements: We thank Todd Westhuis from NYSDOT for his helpful comments on an earlier draft of this paper. Part of this work was supported by September 11th Memorial Program for Regional Transportation Planning administered by the New York Metropolitan Transportation Council (NYMTC) and University Transportation Research Center (UTRC) Region 2. Any opinions, findings, and conclusions expressed in this paper are those of the authors.

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Noname manuscript No.
(will be inserted by the editor)

Dynamic User Equilibrium Model for Combined Activity-Travel Choices Using Activity-Travel Supernetwork Representation

Gitakrishnan Ramadurai · Satish Ukkusuri

the date of receipt and acceptance should be inserted later

Abstract Integrated urban transportation models have several benefits over sequential models including consistent solutions, quicker convergence, and more realistic representation of behavior. Static models have been integrated using the concept of Supernetworks. However integrated dynamic transport models are less common. In this paper, activity location, time of participation, duration, and route choice decisions are jointly modeled in a single unified dynamic framework referred to as Activity-Travel Networks (ATNs). ATNs is a type of Supernetwork where virtual links representing activity choices are added to augment the travel network to represent additional choice dimensions. Each route in the augmented network represents a set of travel and activity arcs. Therefore, choosing a route is analogous to choosing an activity location, duration, time of participation, and travel route. A cell-based transmission model (CTM) is embedded to capture the traffic flow dynamics. The dynamic user equilibrium (DUE) behavior requires that all used routes (activity-travel sequences) provide equal and greater utility compared to unused routes. An equivalent variational inequality problem is obtained. A solution method based on route-swapping algorithm is tested on a hypothetical network under different demand levels and parameter assumptions.

Keywords Integrated urban transport model · Activity-Travel Networks · Dynamic user equilibrium · Route-Swapping Algorithm

G. Ramadurai
PhD Candidate
Department of Civil and Environmental Engineering
Rensselaer Polytechnic Institute
Tel.: +1-518-2768306
Fax: +1-518-2764833
E-mail: ramadg@rpi.edu

S. Ukkusuri
Assistant Professor and Blitman Career Development Chair
Department of Civil and Environmental Engineering
Rensselaer Polytechnic Institute
Tel.: +1-518-2766033
Fax: +1-518-2764833
E-mail: ukkuss@rpi.edu

1 Introduction

Urban transport modeling involves several dimensions of individual choice including activity participation, location, time of participation, duration, choice of mode, and route. Often the choice models are sequentially applied with feedback: initially, the choice environment is assumed fixed and the individual choices are determined. Subsequently, given the individual choices the choice environment is adjusted. If feedback is involved, the two steps are repeated until the individual choices and the resulting choice environment are in equilibrium. We also refer to this state as converged solution. This process of iteratively solving a sequence of models forms the basis of the four-step urban transportation modeling paradigm.

As opposed to the sequential procedure, several studies have explored integrated choice models particularly with respect to static transport models. Integrated urban transportation models have several benefits over sequential models including consistent solutions, quicker convergence, and more realistic representation of behavior. Static urban transport models have been integrated using the concept of Supernetworks ((Sheffi, 1985); also referred to as Hypernetworks, (Sheffi and Daganzo, 1979, 1980)). However integrated dynamic transport models are less common.

Dynamic traffic assignment models (Peeta and Ziliaskopoulos, 2001) have been developed over the past two decades and have addressed several of the short-comings of the static traffic assignment procedures. In particular, DTA models have increased traffic flow and behavior realism and the explicit modeling of time-varying flows (Friesz et al, 2001; Carey, 2001; Boyce et al, 2001; Zhang and Nie, 2005). These advantages allow DTA to be applied to real-time traffic management, ATIS, and other ITS measures (Mahmassani, 2001; Ben-Akiva et al, 2001). While traditionally DTA models were restricted to determining route choices given an exogenous time-sliced demand matrix, more recently DTA models that capture two choice dimensions - route and departure time choice - have been developed (Friesz et al, 1993; Ran et al, 1996; Huang and Lam, 2002; Wie et al, 2002; Szeto and Lo, 2004; Zhang and Zhang, 2007). To capture behavioral realism better there is a need to consider additional choice dimensions within a dynamic traffic assignment framework.

Initial work toward integrating additional choice dimensions in DTA models are (Abdelghany et al, 2001, 2003). Abdelghany et al (2001) develop dynamic spatial microassignment procedures when the unit of analysis is trip chains instead of trips. However, they do not model additional choice dimensions such as departure time, activity location and duration endogenously. Abdelghany et al (2003) addresses a more general choice problem. They determine the departure time, route choice and the sequence of activities simultaneously.

More recent studies in integrated dynamic models include Lam and Huang (2003); Zhang et al (2005); Kim et al (2006); Rieser et al (2007). Lam and Huang (2003) develop a dynamic equilibrium model considering activity location, route, and departure time dimensions. Their framework, however, assumes the duration of activity participation as exogenous. Capturing activity duration is essential to understand the effect of activity scheduling on traffic congestion. An integrated work activity scheduling and departure time choice model in a network with bottleneck congestion is developed by Zhang et al (2005). However, they consider single activity participation only. A logical extension is to consider multiple activities and activity chaining decisions. This is the focus of the paper by Kim et al (2006). They present an activity chaining model formulated from the perspective of a time use problem with budget constraints. Their model includes a dynamic traffic assignment simulation model to obtain network travel times and an iterative day-to-day dynamic process where activity chains are updated based on the network travel times computed in previous iteration. Whether such an iterative procedure results in consistent solutions and the performance of the solutions compared to more holistic frameworks are interesting research questions that merit attention. Rieser et al (2007) describe a multi-agent simulation (MATSim) that takes individuals complete activity sequence as input. Individual's behavior in terms of their route choice and departure time choice are determined iteratively with a traffic flow simulator. They describe a conceptual framework to extend the MATSim to incorporate activity rescheduling and participation decisions.

In this paper, activity location, time of participation, duration, and route choice decisions are jointly modeled in a single unified dynamic framework referred to as Activity-Travel Networks (ATNs). The proposed integrated framework is motivated by the following considerations: (a) to capture activity demand-supply dynamics in addition to transportation demand-supply dynamics, and (b) to obtain a consistent equilibrium solution across all dimensions of choice. ATNs is a type of Supernetwork where virtual links representing activity choices are added to augment the travel network to represent additional choice dimensions. Each route in the augmented network represents a set of travel and activity arcs. Therefore, choosing a route is analogous to choosing an activity location, duration, time of participation and travel route.

2 ATN Representation and Motivation

ATNs use a network representation where nodes are activity centers that are joined by travel links. Activities are represented by arcs that both originate and terminate in the same node (activity centers). Each activity arc is characterized by a unique activity type and a set of durations. An activity-travel sequence for an individual can be represented as a 'route' that includes both travel and activity arcs. All individuals at the beginning of the model start from 'home' and must participate in a predefined set of activities. All activity-travel sequences that traverses the set of activity arcs in which an individual participates in are considered feasible sequences. The model time frame may be set arbitrarily and is presented in a discrete-time setting. Durations of arc-traversal for travel arcs is always assumed to be a function of flow, while for activity arcs it is assumed fixed. Consistent with rational behavior assumption, each individual chooses the activity-travel sequence that provides the maximum generalized utility. However, modeling the network dynamics at an individual level is computationally intensive. Therefore, we treat all individuals residing in the same 'home' node, who participate in the same set of activities as similar. We accordingly modify the behavioral framework to be consistent with Wardrop's (Wardrop, 1952) equilibrium framework. The behavioral rule adopted is 'all used routes (activity-travel sequences) provide equal and greater utility compared to unused routes'. In other words, at equilibrium no individual can improve her utility by unilaterally changing her travel choice decisions.

A primary motivation of the ATNs representation is to capture the effect of activity and transportation demand-supply dynamics in travel choice decisions. Consider a hypothetical scenario in the double-diamond network shown in figure 1. The network consists of eight nodes: Home node (H), Work node (W), four Non-work activity centers (N1-N4), and two intermediate nodes (I1 and I2). The nodes are connected by thirteen arcs: 3, 4, 10, and 11 are the activity arcs and the rest are travel arcs. Let us call the

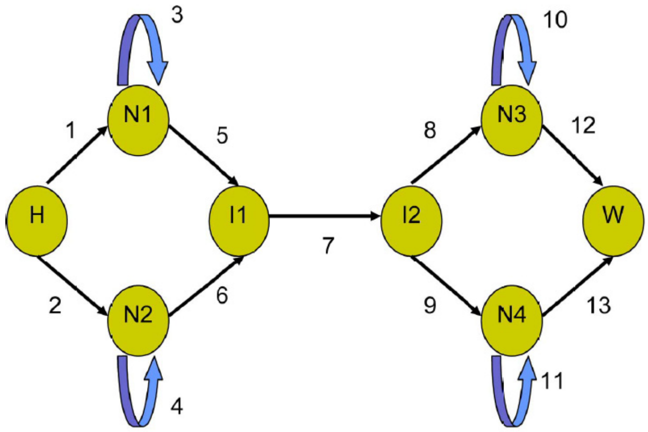

Fig. 1 Example Network - Double Diamond Network

diamond with the home node as the residential neighborhood diamond (R-diamond) and the other as business neighborhood diamond (B-diamond). The total demand for travel from home to work is 100 individuals; all individuals drive alone to work. Further, 50 individuals drive directly from home to work while 50 individuals make a stop to participate in a non-work activity en route to work. All individuals have to arrive at work at the same time, (say) T. All travel arcs have a capacity of 50 vehicles per time unit and free-flow traversal time of one time unit, while the duration of non-work activity participation (which is also the time for traversal of activity arc) is two time units. The utility of participating in the non-work activity is 100 utils (let utils be the unit of measuring utility) while the utility of travel on an arc is -5*(travel time) utils. As mentioned earlier, the travel arcs have fixed capacities: at free-flow a travel arc traversal would fetch -5 utils, while a queuing delay by one time unit would result in a payoff of -10 utils.

There exist two possible activity-chain sequence in the double-diamond network: i) Home to Work, and ii) Home to Non-work activity to Work. The former can be accessed via four different paths while the latter has eight paths - four paths each that visit a non-work activity center in R-diamond and B-diamond. Since the utility of participating in the non-work activity in all four nodes is the same, based on traditional models of utility maximization, they attract equal amount of traffic. Therefore such an assignment model would result in each of the eight paths that pass through the non-work activity having a flow of 50/8. The corresponding total free-flow traversal time is 7 time units (therefore departure time from home node is $(T-7)^{th}$ time unit). For the individuals who drive straight to work, the traversal time is 5 time units; the corresponding flow is divided among the four paths (50/4). However, link 7, with a capacity of 50 vehicles per time unit, has an upstream demand of 75 vehicles at the start of $(T-3)^{th}$ time unit. This leads to delay by one time unit for 25 individuals and a loss in overall utility of 125 utils (assuming there is no late arrival penalty).

On the other hand, if traffic dynamics is incorporated in the assignment model, we would obtain a solution where none of the individuals visit the non-work activity center in the R-diamond. In this case, there is no delay for any of the individuals and the total overall utility is 125 utils more than the previous case. The reason for the difference in utilities is the limited capacity of link 7. Ignoring the traffic flow dynamics, could lead to sub-optimal assignment patterns. Therefore, it is important to consider transportation demand-supply dynamics.

Activity demand-supply dynamics also play a similar important role in individual decisions. Examples include activity centers with access time restrictions, social interaction activities that provide greater utility with increased participation and capacity restrictions in shopping mall check-out counters. Consider the example of capacity restrictions in shopping mall check-out counters: current models that ignore such an activity supply capacity restriction could over-estimate trip-chaining of shopping activity by commuters or under-estimate non-peak hour shopping trips. If in the *double-diamond* network example above, the non-work activity centers located in the B-diamond had the following modified utility specification: 100 utils if flow on arc is less than or equal to 15 individuals, 75 otherwise; then, the corresponding destination choice and traffic assignment model would result in 15 individuals choosing to participate in the non-work activity in B-diamond while 10-individuals choose the R-diamond. ATNs can model the above described as well as several other activity demand-supply dynamics.

3 Operational Framework

Two critical issues to operationalize the ATN framework are flow propagation dynamics and utility function specification. We discuss their implementation details below.

3.1 Dynamics of Flow Propagation

Traffic flow has been modeled at different levels in the past. The most realistic models are disaggregate microsimulation models (Gartner et al, 2001) where behavior of each vehicle on the network is modeled explicitly. On the other hand, macroscopic models (Gartner et al, 2001) describe traffic flow based on relationships between speed, flow, and density. Though microscopic models are more accurate they require greater computation time and lack analytical solutions. More recently mesoscopic models (de Palma and Marchal, 2002; Nokel and Schmidt, 2002) have gained increasing attention especially for studying large-scale networks.

Macroscopic models can be modeled as side constraints to provide approximate, quick solutions and are more suitable for analytical DTA models. Macroscopic models can be further divided into exit flow models, point queue models, and physical queue models. In this study, we use a network level simulation adaptation of the cell transmission model (CTM) (Daganzo, 1994, 1995; Ziliaskopoulos, 2000). The CTM is capable of capturing the effect of spillbacks (physical queue) and shock wave propagation (two-regime flow). Also, the dynamic equilibrium problem considering activities is formulated as a variational inequality (VI) problem. Existing VI solution techniques are based on heuristic searches and require several iterations of network loading step. Therefore, embedding a microsimulation model would require multiple runs of a computationally intensive model and could significantly increase the running time of any algorithm. We present the details of the CTM below.

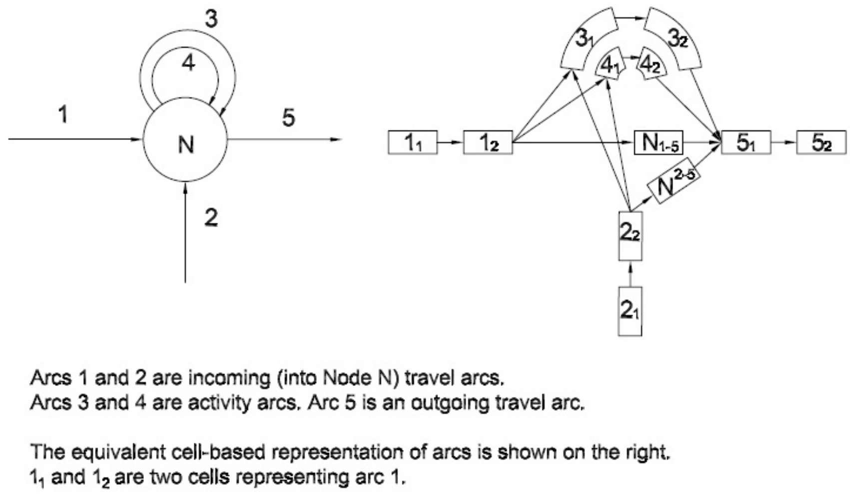

Fig. 2 Cell-based Representation in ATNs

We assume the activity-travel network to be divided into a series of inter-linked cells. Cells represent a segment of a travel link or an activity location. Unlike (Daganzo (1994), Daganzo (1995)) we assume variable cell lengths. As mentioned in Daganzo (1995) this implies a trade-off between computational resource requirements and the level of accuracy of the CTM model to the classic Lighthill-Whitham-Richards (LWR) model - shorter cells can more closely replicate the LWR model but may demand more computational resources. Allowing for variable cell lengths is straight-forward in simulation adaptations of the CTM. Activity cells do not have a physical length; traversal time of activity cells are determined based on the duration of activity participation information contained in the route chosen for travel. The links between cells do not have any physical significance. An example of the cell transmission model representation of activities at a node is shown in Figure 2.

Notation:

Let,

$x_t^{i,r}$ is the number of vehicles following route r in cell i that entered the cell at $(t-t_f^i)^{th}$ time interval or earlier. In the case of activity cells, this represents the number of vehicles that have 'resided' in cell i for a time period equal to the duration of activity participation.

$x_t^i = \sum_{\forall r} x_t^{i,r}$

X_t^i: the total number of vehicles in cell i at time t.

$y_t^{i,r}$: flow on route r, out of cell i at time t.

$y_t^i = \sum_{\forall r} y_t^{i,r}$

N^i: Number of vehicles that can be accommodated at jam density for cell i.

Q^i: Maximum flow capacity out of cell i.

We assume N and Q are time invariant and therefore drop the time subscripts in their representation.

In the discussion below, cells j and k are assumed to be immediately downstream of cell i and cells g and h are immediate upstream of cell i.

Finally, P^g represents the fraction of flow from cell g that enters the downstream merge cell, i. The sum over all such fractions is one (in our illustration we assume two cells - g and h - merge into cell i. Therefore $P^g + P^h = 1$).

Flow propagation is achieved by repeatedly solving three sets of equations - first set of equations determine the outflow from a cell (y) between time-step *t-1* to t, second set of equations determine the individual route break-ups and the final set determine the current cell occupancy ($x_t^{i,r}$) based on past occupancy, inflows, and outflows.

y_t^i is determined from the following equations:

For activity cells: $y_t^i = x_t^i$.

For ordinary travel cells: $y_t^i = \min(x_{t-1}^i, Q^i, N^j - X_{t-1}^j)$.

For cells that merge into a single cell, several cases arise. We deal with each below:

Case 1: If $\min(x_{t-1}^g, Q^g) + \min(x_{t-1}^h, Q^h) > (N^i - X_{t-1}^i)$

Case 1a: If $\min(x_{t-1}^g, Q^g) > P^g(N^i - X_{t-1}^i)$ and $\min(x_{t-1}^h, Q^h) > P^h(N^i - X_{t-1}^i)$, then $y_t^g = P^g(N^i - X_{t-1}^i)$ and $y_t^h = P^h(N^i - X_{t-1}^i)$.

Case 1b: Else If $\min(x_{t-1}^g, Q^g) \leq P^g(N^i - X_{t-1}^i)$, then $y_t^g = \min(x_{t-1}^g, Q^g)$ and $y_t^h = (N^i - X_{t-1}^i) - y_t^g$.

Case 1c: Else, $y_t^h = \min(x_{t-1}^h, Q^h)$ and $y_t^g = (N^i - X_{t-1}^i) - y_t^h$.

Case 2: Else, $y_t^g = \min(x_{t-1}^g, Q^g)$ and $y_t^h = \min(x_{t-1}^h, Q^h)$.

For diverge cells, x_t^i is split into two parts x_t^{i,r_j} and x_t^{i,r_k} such that $x_t^{i,r_j}(x_t^{i,r_k})$ contains all vehicles that take cell j (k) next. This is determined based on the next cell in route r. The outflow into each of the two diverge links may be determined similar to an ordinary cell with route specific cell occupancies x_t^{i,r_j} and x_t^{i,r_k} instead of x_t^i.

The second set of equations determine the flow on each route r: $y_t^{i,r} = \dfrac{y_t^i}{x_{t-1}^i} x_{t-1}^{i,r}$.

The third step of determining current occupancy follows from $x_t^{i,r} = x_{t-1}^{i,r} + y_t^{j,r} - y_t^{i,r}$.

The reader is referred to Lo and Szeto (2002) for a detailed discussion on obtaining average travel times from the CTM simulation. An additional step required in the current model is to deduct activity participation durations from the computed travel times.

3.2 Utility Function Specification

The next critical step in the ATN framework is the utility function specification. The focus of the present paper is not on estimating utility function form or parameters. We assume reasonable functional forms and parameter values to illustrate the ATN framework. However, accurate estimation of utility function form and parameters is an important issue that needs further investigation in the future.
Let,
A^c be the set of all possible activity combinations.

R_{od}^a : Set of routes from origin o to destination d containing activity arcs α^A such that they traverse all activities in activity combination $a \in A^c$. r is a route that belongs to the set R_{od}^a. Each route r represents a set of travel and activity arcs. Therefore choosing a route r, results in the choice of activity location, duration, time of participation and travel route.

$U_{od}^{a,r}$ denotes utility derived by individuals departing from o and reaching d, participating in activity chain $a \in A^c$ using route r.

$h_{od}^{a,r}$: Path flow from o to d, participating in activity chain combination $a \in A^c$ using route r.

The temporal dimension in dynamic traffic assignment models (such as departure or arrival time index) is not associated with the above definitions since all individuals are always traveling on the network or participating in an activity.

Similar to Lam and Huang (2003), we assume an additive specification for the above utility expression.

$$U_{od}^{a,r} = U^a(r) - U^{trav}(r) \tag{1}$$

where, $U^a(r)$ is the utility derived from participating in activity combination $a \in A^c$ and is a function of route r. $U^a(r)$ can be represented as the sum of utilities derived from traversing each activity arc α_a in route r.

$$U^a(r) = \sum_{\forall \alpha^A \in r} U_\alpha(r, f) \tag{2}$$

where, f is the flow in activity link. In general, utility derived from activity participation may be assumed to be a function of type and duration of activity, time of participation, location of activity with respect to the origin/destination of flow on route r, and the total flow on activity link α^A.

$U^{trav}(r) = \beta * TT(r)$ is the disutility from travel on route r, where β is a parameter to convert travel-time into utility units and $TT(r)$ is the total travel time on route r.

4 Mathematical Formulation of ATNs

4.1 Dynamic User Equilibrium Conditions

We can now express the DUE conditions as follows:

$$U_{od}^{a,r} = \begin{cases} = \overline{U}_{od}^{a} \text{ if } h_{od}^{a,r} > 0 \\ \\ \leq \overline{U}_{od}^{a} \text{ if } h_{od}^{a,r} = 0 \end{cases} \quad \forall \text{o,d}, a \in A^c, \text{and r}(: r \in R_{od}^{a}) \tag{3}$$

Subject to the condition that flow on network should satisfy demand. This is expressed as:

$$\sum_{\forall r \in R_{od}^{a}} h_{od}^{a,r} = \sum_{\forall i_h \in (o,d)} \zeta_{i_h}^{a} \quad \forall a \in A^c, o, d \tag{4}$$

where, $\zeta_{i_h}^{a} = \begin{cases} 1 \ldots \text{ if activity combination } a \in A_{i_h} \\ 0 \ldots \text{ otherwise} \end{cases}$

$\overline{U}_{od}^{a}$ is the maximum utility derived by individuals departing from o and reaching d, participating in activity combination $a \in A^c$ using route r.

DUE conditions, however, are not always satisfied in capacitated networks (Szeto and Lo, 2006). Discontinuities in travel time or utility functions could result in non-existence of solutions. These discontinuities could arise from time discretization or due to capacity restrictions in the network. In capacitated networks it is possible that packets of flow are broken because of the lack of available capacity downstream. Any discrete-time model in capacitated networks exposes itself to the above drawback. Further study is required to understand the properties of DUE in discrete-time capacitated network models.

4.2 Equivalent variational inequality formulation

The above DUE conditions can now be formulated as an equivalent VI problem. Formulating DUE conditions as a variational inequality have been proposed by several authors in the past including Friesz et al (1993); Ran and Boyce (1996); Ban et al (2008).

$$\sum_{\forall a \in A^c} (\mathbf{h}^a - \hat{\mathbf{h}}^a)^T U^a(\mathbf{h}) \geq 0 \quad \forall \mathbf{h}^a \in H^a \text{ and } \forall a \in A^c \tag{5}$$

where,
H^a is the set of feasible route flows traversing all activities in activity combination a, given by (4),
$\mathbf{h}^a$ is the vector of route flows $\in H^a$,
$\hat{\mathbf{h}}^a$ is the vector of route flows that satisfy the DUE condition in equation 3, and
U^a is a vector whose each element is given by $U_{od}^{a,r} - \overline{U}_{od}^{a}$.

4.3 Solution Approach

The utility derived from traversing the activity-travel sequence represented by route r, expressed as the sum of utility derived from participating in activities and the disutility from travel, is assumed to be a monotone decreasing function of flow on route r. Therefore, a route-swapping algorithm (Lam and Huang, 2003; Szeto and Lo, 2006; Nagurney and Zhang, 1997) is adopted to obtain solutions to the VI problem shown in (5). The detailed algorithm is presented below:

Step 0: Initialize. Set iteration counter $i = 0$.
Choose an initial feasible vector of flows $\mathbf{h}(i)$.

Step 1: Computation. Load flow $\mathbf{h}(i)$ and compute travel times $\mathrm{TT}(r)$ using the Cell-based transmission model.

Compute utilities $U_{od}^{a,r}$ using (5) $\forall\ r,\ a,\ o,\ d$.

Set $\overline{U}_{od}^{a} = \max\limits_{\forall r \in R_{od}^{a}} U_{od}^{a,r}\ \ \forall\ \ a,\ o,\ d.$

Step 2: Update flows. Set $\hat{R}_{od}^{a} = r \in R_{od}^{a} : U_{od}^{a,r} = \overline{U}_{od}^{a}$.
For ever activity combination $a \in A^{c}$,

$$h_{od}^{a,r}(i+1) = \max(0, h_{od}^{a,r}(i) + \rho h_{od}^{a,r}(i)(U_{od}^{a,r} - \overline{U}_{od}^{a}))\ \forall r \in R_{od}^{a} \backslash \hat{R}_{od}^{a}$$

$$\Sigma_{od}^{a} = \sum_{\forall r \in R_{od}^{a} \backslash \hat{R}_{od}^{a}} (h_{od}^{a,r}(i) - h_{od}^{a,r}(i+1))\ \ \forall\ \ a,\ o,\ d.$$

$$h_{od}^{a,r}(i+1) = h_{od}^{a,r}(i) + \frac{\Sigma_{od}^{a}}{|\hat{R}_{od}^{a}|}\ \ \ \forall r \in \hat{R}_{od}^{a}\ \ \ \forall\ \ \ r,\ a,\ o,\ d.$$

ρ is a scale parameter.

Step 3: Check for convergence. Compute $\pi = \sum\limits_{\forall r,a,o,d} (U_{od}^{a,r} - \overline{U}_{od}^{a})\, h_{od}^{a,r}$ and $\hat{\pi} = \sum\limits_{\forall r,a,o,d} \overline{U}_{od}^{a}\, h_{od}^{a,r}$.
If $\frac{\pi}{\hat{\pi}} < \epsilon$ then terminate. ϵ is a convergence tolerance value.
else, i = i + 1; Go to Step 1.

Nagurney and Zhang (1997) use the route-swapping algorithm (referred to as Euler's method) for the static traffic assignment problem. They show that the Euler's method converges only when the link costs are strictly monotone increasing. However when implementing the algorithm for path based formulations they reported that the algorithm did not converge in their limited trials. Other studies (Lam and Huang, 2003; Szeto and Lo, 2006) also report lack of smooth convergence in their implementations of the algorithm. Therefore, the route swapping algorithm has convergence issues when implemented in path-based formulations. In the numerical trials we test different scenarios and report under what conditions the route swapping algorithm appears to provide consistent DUE solutions to a test problem. Recently, Mounce and Carey (2008) report important convergence properties for the route-swapping algorithm. However, the particular mathematical properties required for convergence are not satisfied here; in particular, we have a discrete system with discontinuities in the cost function and its gradients due to the capacitated network flow model assumption.

Two important components in the above algorithm are the convergence expression and the scale parameter ρ. Traditionally, the literature has utilized a convergence check based on flow changes between two iterations (Nagurney and Zhang, 1997; Abdelghany et al, 2003). However, since path flows are not necessarily unique (assuming the solution exists), using flow to determine convergence could lead to infinite loops in the algorithm. A better convergence measure is the utility difference of used paths in successive iterations. However, a direct comparison of utilities in used paths could lead to the algorithm converging to a non-equilibrium solution. The expression used above overcomes this problem by comparing the utility on all used paths to the maximum possible utility. This ensures that the algorithm converges only when all used paths have an utility that is close the maximum possible utility at equilibrium. If there are no equilibrium solutions, the algorithm will not terminate. A reasonable upper limit on the number of iterations is required to ensure the algorithm does not loop infinitely. Consequently, if the algorithm terminates after reaching the maximum number of iterations, the solution must be checked to see if it has converged to an equilibrium solution or not.

Nagurney and Zhang (1997) provide conditions for the scale parameter ρ under which the algorithm converges to an equilibrium solution. Lam and Huang (2003) show that these conditions allow for local stability if the cost functions are not strictly monotonic. However, in discrete time capacitated networks such as the one dealt with in this paper, the utility functions are likely to be discontinuous with sudden jumps and falls. This adds to the complexity of the algorithm convergence and no theoretical properties for the value of the scale parameter ρ exist. This problem is also reported in Szeto and Lo (2006). We test different values for the scale parameter and draw limited insights.

5 Results from an Example Network

We demonstrate the application of the ATN framework and the proposed solution algorithm on an example network. The example network considered is the double-diamond network presented earlier. The equivalent cell-based representation of the network is shown in figure 3. Free-flow traversal time, maximum flow capacity, and number of vehicles at jam density for square (rectangular) cells are 1 minute, 1000 vehicles/minute, and 3500 vehicles (3 minute, 1000 vehicles/minute, and 10500 vehicles).

There are two possible activity chains - home to work (H-W) and home to non-work to work (H-NW-W). All individuals depart from home; departure times are set at every fifth minute starting from (and including) 7:00 AM. The preferred work arrival time is 8:00 AM. The possible non-work activity durations are 5, 10, 15, 20, 25 and 30 minutes. Since free-flow travel time is 25 minutes, 7:35 AM (7:30 AM) is assumed to be the latest departure time for individuals participating in H-W (H-NW-W) activity chain. There are 4 (8) travel route options available for activity chain H-W (H-NW-W). Therefore, for the H-W activity chain there are (4x8=) 32 route options numbered from zero to thirty one [1], while there are (8x27=) 216 route options for the H-NW-W activity chain [2] numbered from thirty two to two hundred and forty seven. Three demand levels are analyzed. Low, medium, and high demand representing 750, 3750, and 7500 individuals participating in each activity chain combination are considered.

[1] Includes departure time options 7:00, 7:05, ... 7:35 and 4 travel routes

[2] Including 4 locations, 2 travel routes, and 6 non-work activity duration options for individuals departing at 7:00 AM and 7:05 AM, 5 for 7:10 AM departures, 4 for 7:15 AM departures and so on - totaling to 27 activity duration combinations.

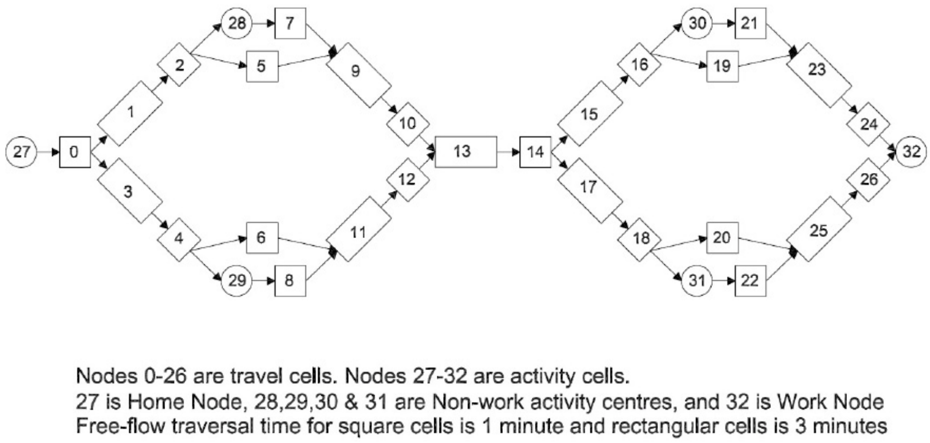

Fig. 3 CTM Representation of Double-Diamond Network

The utility profiles for the different activities are presented in Figure 4. Home stay is rewarded with 100 utils/min. The utility derived from non-work activity participation are identical for all four locations; they depend on duration of participation only. Starting at zero utils for zero minutes duration, the utility derived from every additional minute increases linearly to a maximum of 125 utils/min for duration of 15 minutes and then drops linearly to 0 utils/min at 30 minutes of activity participation. The utility is assumed to be independent of time of participation or flow on activity arc. The preferred arrival time at work is 8:00 AM. Early arrivals are penalized at the rate of -50 utils/min and late arrivals are penalized at -150 utils/min. Travel disutility is -100 utils/min of travel.

Three different scale parameter ρ values were tested. ρ was first assigned to an initial value (*1/n*) and then progressively reduced to 1, 1/2, 1/3, 1/4... of the initial value. Each value was held constant for n iterations. For example when $n = 10$, the initial value of ρ is set to 0.1 for the first 10 iterations, then reduced to 0.05 (= 0.1/2) for iterations 11 through 20, then reduced to 0.033 (= 0.1/3) for iterations 21 through 30 and so on. Three different values of n (100, 1000, and 10000) were tested.

In terms of the overall results, the high demand case did not converge to an equilibrium solution. It is likely that there is no equilibrium solution for the high demand case, however there is no theoretical method to check for equilibrium existence in discrete time systems with cell-transmission based traffic flow model. The inverse of the initial value of the scale parameter, $n = 100$ converged immediately (to a non-equilibrium solution) and have not been included in the results presented below.

The convergence measure for $n = 1000$ and $n = 10000$ are presented in Figures 5 and 6. As can be seen from the figures, the low demand case converges to an equilibrium very smoothly. There are a few spikes in the medium demand case (for both values of n), and even more spikes for the high demand case. Further, the rate of convergence is faster for the lower value of n (higher value of scale parameter ρ). This is consistent with the result in Szeto and Lo (2006).

As mentioned earlier the high demand case did not converge to any equilibrium solution. However, in general the algorithm progressed such that the maximum flow

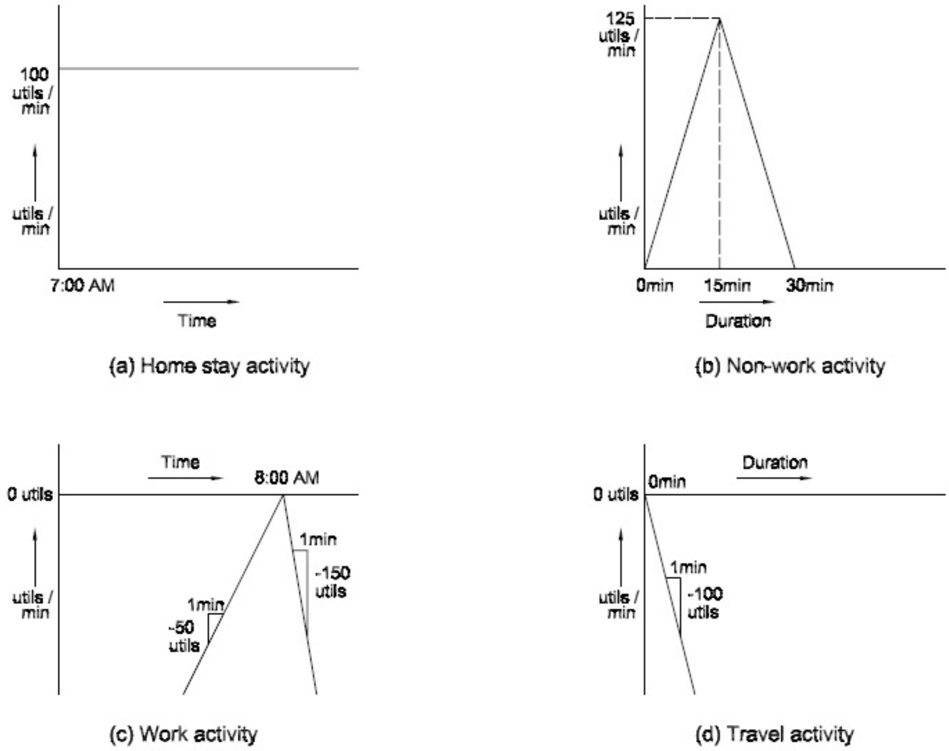

Fig. 4 Cell-based Representation in ATNs

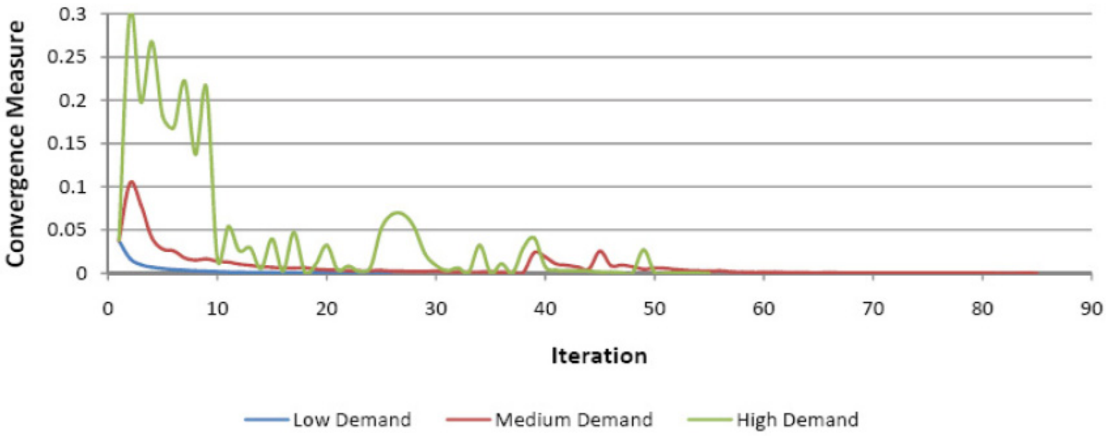

Fig. 5 Convergence of Route-Swapping Algorithm, n=1000

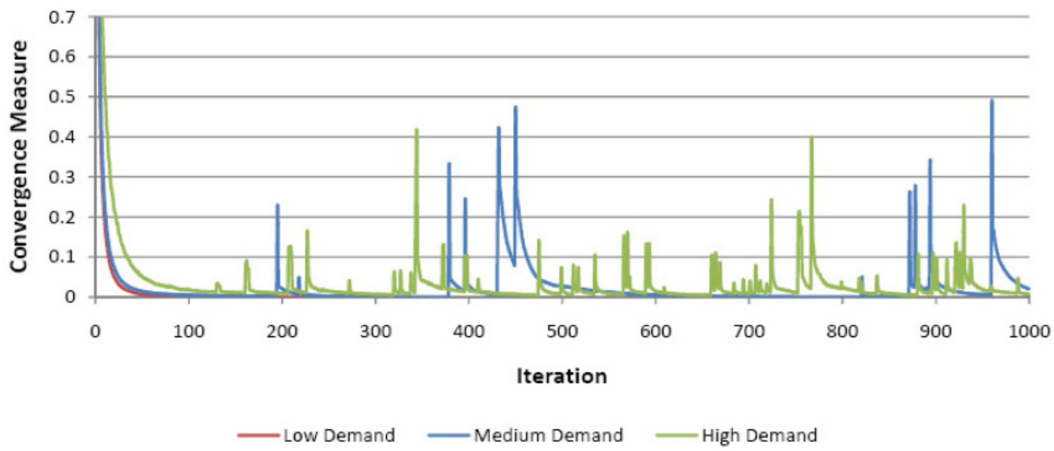

Fig. 6 Convergence of Route-Swapping Algorithm, n=10000

was loaded on to the route with maximum utility. This result was observed only for $n = 10000$. Even though the rate of convergence is faster for $n = 1000$, the solution obtained for the high demand case was poor. That is the maximum flow was not among routes with maximum utilities.

The flow values and corresponding average utilities for both H-W and H-NW-W activity chains for all three demand levels are presented in Tables 1 and 2. The solutions correspond to converged values for the low and medium demand cases and 100,000 iterations for the high demand case. The value of n used here is 10000. When the route swapping algorithm converges, it is straightforward to verify if the solution is an equilibrium solution. Since the algorithm provides flow and utility values for all routes, one could check if all the flow are present on the routes that provide the maximum possible utility. On the contrary, if there exist routes with greater utility values than any other route with a positive flow, the solution is not an equilibrium. Proving the theoretical correctness of whether the route swapping process will always converge to an equilibrium solution in discrete systems with discontinuous utility functions remains an important question for future research.

From Tables 1 and 2, we can observe that the low and medium demand cases have converged to an equilibrium solution since almost 99.9% of the flows are in routes that provide maximum utility. In the high demand case, however, the solution obtained after 100,000 iterations is not an equilibrium solution. Almost all used paths in the H-W activity chain and 98.3% of flow in the H-NW-W activity chain have equal and maximum utility values. However, about 1.7% of the flows in the H-NW-W activity combination have utilities lesser than the maximum possible utility. While equilibrium existence cannot be ruled out completely, a reason for the lack of convergence could be because of the capacity restrictions. Any change in flow on particular routes could translate to significant delays due to discontinuities arising from the capacity restrictions. A numerical test of positively perturbing the flow on the maximum utility path indicated a significant drop in the corresponding utility value. The solution obtained here may be considered a Nash equilibrium since any unilateral shift did not improve the payoff of any of the users. However, it is clearly not a dynamic user equilibrium solution. Further, the presence of unequal flows even among symmeteric paths - 31, 7, 23 and 15 for example - (notwithstanding approximations arising from sequential flow propagation steps) indicates the possibility of non-uniqueness of solutions in terms of path flows in DUE.

An intuitive result obtained in the example in Section 2 is also corroborated here. The intuition is that individuals would prefer participating in the non-work activity at the location in B-diamond instead of R-diamond to avoid queuing at the bottleneck link joining the two diamonds. We observe this in Table 2. The flows on routes shown in the table accounts for 98.3% of equilibrium flow and all individuals contributing to this flow prefer the non-work activity center in B-diamond (cells 30 and 31 shown in Figure 3). In terms of durations, individuals in the H-W activity chain prefer departing as late as possible (7:35 AM) while few depart at 7:30 AM. For H-NW-W activity chain, 7:30 AM departure and a non-work activity duration of 5 minutes is the most preferred.

In summary, the route-swapping algorithm performed reasonably well. Even for high demand cases when there is no equilibrium solution the algorithm approaches a 'good' solution that ensures more flow on higher utility paths. However as demand increases the convergence is not smooth - there are several spikes as seen in Figures 5 and 6. A reason for the spikes may be because of the discrete time capacitated network considered here - a small shift in flow to some paths may result in substantial delays and disutilities. Also, the convergence value was found to depend on the ρ parameter though clear patterns were not obtained. These remain important issues for future studies.

Table 1 Equilibrium flows for activity-chain H-W

Route Id	Low Demand		Medium Demand		High Demand		Home Stay
	Flow	Utility	Flow	Utility	Flow	Utility	Duration
31	197.72	1050	802.9	796.71	1861.9	243.49	35 min
7	172.86	1050	1039.34	796.71	1759.29	244.93	35 min
23	153.16	1050	1072.82	796.71	1768.86	244.38	35 min
15	226.21	1050	834.94	796.72	1712.35	244.04	35 min
6	0	300	0	300	89.74	239.43	30 min
22	0	300	0	300	115.47	239.42	30 min
30	0	300	0	300	98.35	239.42	30 min
14	0	300	0	300	94.04	239.43	30 min
Other routes (24 nos.)	0	< 300	0	< 300	0	$\leq$ -450	$\leq$ 25 mins

6 Summary and Further Work

In this paper, an integrated formulation to obtain equilibrium solutions across multiple dimensions of travel choice is presented. The formulation is based on a Supernetwork representation referred to as Activity-Travel Network (ATN) representation. In ATN representation, nodes are activity centers that are joined by travel links. Activities are represented by arcs that both originate and terminate in the same node (activity centers). An activity-travel sequence for an individual can be represented as a 'route' that includes both travel and activity arcs. The 'route' choice in an ATN results in simultaneous determination of activity location, time of participation, duration, and route choice decisions.

Table 2 Equilibrium flows for activity-chain H-NW-W

Route Id	Low Demand		Medium Demand		High Demand		Home Stay	NW Activity	NW Activity
	Flow	Utility	Flow	Utility	Flow	Utility	Duration	Duration	Location
220	96.3	654.2	801.6	472.7	783.4	115.2	30 min	5 min	31
139	191.2	654.2	611.5	472.6	756.3	116.2	30 min	5 min	30
112	148.1	654.2	708.8	472.7	711.4	116.3	30 min	5 min	30
247	64.1	654.2	900.7	472.6	705.9	115.2	30 min	5 min	31
214	0	508.3	134.6	472.7	413.2	116.1	15 min	20 min	31
133	0	508.3	182.9	472.7	406.5	116.4	15 min	20 min	30
241	0	508.3	227.2	472.7	402.3	116.1	15 min	20 min	31
106	0	508.3	179	472.7	455.7	116.4	15 min	20 min	30
136	0	487.5	0.1	451.9	349.2	116.5	20 min	15 min	30
109	0	487.5	0.1	451.9	345.2	116.5	20 min	15 min	30
244	0	487.5	0.1	451.9	346.8	116.3	20 min	15 min	31
217	0	487.5	0.1	451.9	394.7	116.3	20 min	15 min	31
138	0	466.7	0	466.7	313.2	116.3	25 min	10 min	30
219	0	466.7	0	466.7	313.1	116.2	25 min	10 min	31
246	0	466.7	0	466.7	313.1	116.2	25 min	10 min	31
111	0	466.7	0	466.7	317.5	116.3	25 min	10 min	30
193	77.3	654.2	0.4	419.8	11.8	66.6	30 min	5 min	29
85	74	654.2	0.8	391.6	11.2	46.5	30 min	5 min	29
166	50	654.2	0.4	419.8	13.4	66.4	30 min	5 min	28
58	48.3	654.2	0.8	391.6	10.88	46.3	30 min	5 min	28
Other routes (196 nos.)	0	≤ 300	0	≤ 466.7	$< 1.7\%$	≤ -69.8	Varies	Varies	Varies

The proposed integrated framework allows (a) to capture activity demand-supply dynamics in addition to transportation demand-supply dynamics, and (b) to obtain a consistent equilibrium solution across all dimensions of choice. A rigorous mathematical and operational framework for ATNs based on dynamic user equilibrium behavior with an embedded cell-based transmission traffic flow model was presented. The equivalent variational inequality problem was obtained. A solution method based on route-swapping algorithm is proposed and demonstrated on an example network.

Several open issues merit further investigation: first, we need to derive the properties such as solution existence and uniqueness of the variational inequality problem. Second, numerical or analytical results on convergence properties of solution algorithms need to be developed. This would depend on the utility function specification and the traffic flow dynamic model among other factors. Third, more sophisticated representation for the utility function and the activity-travel choice mechanism can be explored. Finally, the solution algorithm presented here did not converge smoothly for higher demand values (more congested cases). While this can be improved by adopting a finer resolution of time discretization, it leads to increase in the number of route alternatives. Algorithms that obviate the need for route enumeration can solve the problem significantly faster. Faster solution algorithms will allow the Activity-Travel Network framework to be adopted in real-time traffic management applications even in large scale networks.

Acknowledgements We thank two anonymous referees for their comments on an earlier version of the paper. Parts of this work were supported by September 11th Memorial Program for Regional Transportation Planning administered by the New York Metropolitan Transportation Council (NYMTC) and the Emerging Scholars Grant from the University Transportation Research Center, New York City. Any opinions, findings, and conclusions expressed in this paper are those of the authors.

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B-Dynamic: An Efficient Algorithm for Dynamic User Equilibrium Assignment in Activity-Travel Networks

Gitakrishnan Ramadurai, Satish Ukkusuri

May 29, 2009

In the previous paper activity location, time of participation, duration, and route choice decisions were jointly modeled in a single unified representation framework referred to as Activity-Travel Networks (ATNs). ATNs are Supernetworks (Sheffi 1985) where activities are represented as links in the network. A major hurdle for extending the Supernetwork concept to dynamic networks considering activities is that the resulting multi-dimensional dynamic choice problem leads to combinatorially increasing choice dimensions. Therefore existing algorithms that depend on path enumeration such as route-swapping algorithm are difficult to implement even for moderately sized networks.

In this paper an alternative algorithm that does not require path enumeration is presented. The algorithm is a novel extension of Algorithm B (Dial 2006) to dynamic networks and hence referred to as Algorithm B-Dynamic. The rest of the chapter is organized as follows: the literature in static traffic assignment algorithms is reviewed first followed by algorithms for solving dynamic traffic assignment (DTA), and DTA considering additional choice dimensions. The reviews are brief and illustrative; not comprehensive. The next section describes Dial's Algorithm B in detail which provides the platform for Algorithm B-Dynamic that is described next. Numerical examples are then presented to illustrate both the method of the algorithm and its' capabilities.

1 Literature Review

1.1 Algorithms for Static Traffic Assignment

The well accepted static traffic assignment formulation of Beckmann et al. (1956) has been solved using several different techniques. Broadly the algorithms may be termed as link based and path based solution algorithms. Link based solution algorithms store only the flows that use each link while path based solution algorithms store the flow in each path separately. The link flow is an aggregation of path flows that traverse a link. Since the number of links in a network are much lesser than the number of paths between any two O-D pairs, the link based solution algorithms are more memory efficient. For this reason, link based methods have been more widely accepted and implemented in practice.

Two of the most common link based solution algorithms for static traffic assignment problem are the Method of Successive Averages and the Frank-Wolfe method (Sheffi 1985). These algorithms iteratively improve the link flow solutions by solving a linearized problem of the original traffic assignment. While the major advantage of these solution algorithms is their lean memory requirements, it is well known they have poor convergence properties. Beyond a level of accuracy, the tail of the convergence graph flattens out (Bar-Gera 1999).

In contrast to the link based solution algorithms, path based solution algorithms do not suffer from poor convergence and have been shown to obtain more precise solutions in lesser time. Path based algorithms were among the original methods proposed to solve the static traffic assignment problem. The overall concept of path based methods is to shift flows from more expensive paths to less expensive paths till all the paths are in equilibrium. The method was originally suggested by Dafermos (1968) and Dafermos and Sparrow (1969). Bertsekas (1976) proposed a method of shifting flow between independent path segments which was implemented in Jayakrishnan et al. (1994).

All of the above path based methods required the storage of paths across iterations. As mentioned earlier, path storage requires substantial memory and in large networks may be infeasible. Dial (2006) proposed an algorithm (Algorithm B) that does not require explicit storage of paths. He makes use of acyclic sub-networks referred to as bushes to perform flow shifts for equilibration. Dial's Algorithm B is fast since it is path-based and at the same time is efficient since it does not require paths to be stored. To date, Algorithm B is the fastest algorithm available for solving the static traffic assignment problem. Bar-Gera (1999) developed an origin based algorithm that is similar to Algorithm B. Bar-Gera (1999) differentiates his method as being origin based and distinct from the link based and route based methods. Dial (2006) reports that Algorithm B's running times are better than Bar-Gera's origin based algorithm.

The algorithm presented in this chapter (B-Dynamic) is similar in principle to Dial's Algorithm B. However, the current problem context is more complex since we consider traffic dynamics and multiple decision dimensions. Dial's Algorithm B and the proposed B-Dynamic algorithm is discussed in detail later. Other dynamic traffic assignment algorithms are reviewed below.

1.2 Algorithms for Dynamic Traffic Assignment

Unlike static traffic assignment, the area of DTA does not have a single well defined problem statement. The primary goal of DTA is to improve on traffic realism when compared to static assignment. Traffic realism may be improved in several different ways and this has resulted in several alternate DTA paradigms. Extensions of static traffic assignment framework by adding the temporal dimension and time varying link performance functions form the lower spectrum (in terms of traffic flow realism) of DTA models commonly referred to as analytical models. On the other hand, more realistic representation of traffic flow has been achieved by adopting simulation frameworks. Here, the discussion is restricted to algorithms for solving simulation based DTA models.

Algorithms implemented in the following models are discussed - DYNASMART (Mahmassani 2001), DynaMIT (Ben-Akiva et al. 2001), and Dynameq (Florian et al. 2008).

DYNASMART (Mahmassani 2001) (DYnamic Network Assignment-Simulation Model for Advanced Road Telematics) is a simulation assignment model that combines a microscopic level of representation of individual travel decisions with a macroscopic description of traffic flow. A time-dependent origin-destination (O-D) matrix is assumed to be given and based on assignment rules the demand is assigned to routes. Several alternative assignment rules are modeled including system optimal (SO) objective, user equilibrium (UE) objective, boundedly rational path switching, and a current best path assignment when a traveler can consult instantaneous traffic information while at the origin. Traffic simulation is modeled at a macroscopic level but at the same time individual or groups of vehicles are tracked by the simulation. Link movements are modeled using a modified Greenshield speed-density relationship and node transfers are used to compute delays when transferring between links. The framework may be described as a path based framework with a multi-user class K-shortest path algorithm providing available paths for processing. The solution algorithm is iterative with all-or-nothing assignment employed for drivers with SO and UE objectives, and route switching rules for boundedly rational users. Given, the mixed behavior models included in the simulation there is no single overarching analytical formulation or equilibrium criteria that can be used to describe the framework. However, it should be possible to isolate the different behavioral models to develop equilibrium frameworks.

DynaMIT (Ben-Akiva et al. 2001) (Dynamic Network Assignment for the Management of Information to Travelers) uses demand and supply simulators in real-time dynamic traffic assignment to generate consistent user optimal guidance information. DynaMIT combines demand and supply simulation tools such that they can be used for both prediction of future conditions and estimation of current state as well as remain flexible enough to simulate at different levels of aggregation. For example, individual responses to information needs to be captured at a disaggregate level while OD estimation is performed at an aggregate level. Traffic flow is modeled as a hybrid effect resulting from capacity restrictions on the roadways, deterministic queuing at bottlenecks, and macroscopic speed-density relationships. Traffic simulation is performed in two phases. First, the update phase, macroscopic parameters such as speed and density are updated. Second, the advance phase vehicles are advanced to new positions at a microscopic level. Since the behavioral framework adopted is at a disaggregate level that is based on discrete choice models DynaMIT does not lend itself to equilibrium based DTA analysis.

Dynameq (Florian et al. 2008) is a simulation based dynamic traffic assignment model that incorporates user equilibrium behavior. Traffic flow is modeled as a discrete-event procedure that moves individual vehicles similar to microscopic models (car following, lane changing and gap acceptance effects are captured). Time-dependent OD matrices are assumed as input. Paths are initialized based on an incremental assignment technique and updated in subsequent iterations with a new set of dynamic shortest paths. After a pre-specified number of iterations, only used paths are retained and no new paths are updated. The algorithm employs the method of successive averages (MSA) to update path flows between iterations.

As described above several variations of simulation based DTA models exist in the literature. Few of them do not consider equilibrium assignment since existence of and convergence to an equilibrium solution cannot be guaranteed. The emphasis in all the studies has been more on describing traffic flow in greater detail. The algorithms invariably require path storage and employ iterative methods to update path flows.

1.3 Algorithms for DTA Models With Additional Choice Dimensions

The following studies that incorporate DTA with additional choice dimensions are reviewed in this sub-section: Abdelghany et al. (2001), Abdelghany and Mahmassani (2003), and Lam and Huang (2003).

Abdelghany et al. (2001) develop a model where the unit of analysis is trip chains instead of trips. They develop an iterative simulation-assignment algorithm that is consistent with UE behavior. The simulation model embedded in the framework for traffic flow propagation is DYNASMART. An interesting variation employed in this paper is to assign each segment in a trip chain independently using shortest path for each segment. Abdelghany et al. (2001) argue that the alternative of allowing individuals to optimize over the entire trip chain could result in individuals choosing longer routes in one segment to save time on another segment. In contrast, in this chapter, the entire trip chain including activity participation is modeled as an analogous route choice problem. Therefore, individuals optimize over their complete activity-travel sequence. Abdelghany et al. (2001) use MSA to update route flows and use a convergence criterion based on difference in number of vehicles assigned to a path over successive iterations. It must be noted that solution obtained with such a convergence measure may not necessarily satisfy equilibrium conditions.

Abdelghany and Mahmassani (2003) solve the DTA problem considering departure time, route choice, and sequence of activities simultanesouly. The model is formulated as a stochastic dynamic user equilibrium problem and solved using an iterative method that employs MSA. The algorithm employs K-shortest travel time paths between any two stops in the trip chain. Subsequently a superset of all combinations of these shortest paths are enumerated. A stochastic networking loading method is employed as opposed to the all or nothing assignment in Abdelghany et al. (2001). The convergence criteria is similar to Abdelghany et al. (2001) and hence suffers from the drawback mentioned above. Further since the algorithm requires path enumeration (albeit restricted to combination of K-shortest paths in each trip segment) it is anticipated the algorithm performs slower compared to Abdelghany et al. (2001). Possible support for this is that the test problem solved in Abdelghany and Mahmassani (2003) is much smaller compared to the test problem in Abdelghany et al. (2001).

Lam and Huang (2003) develop combined activity/travel choice models considering activity location, route, and departure time dimensions. They employ a route swapping algorithm to solve the problem similar to the method described in the previous chapter. As mentioned in the previous chapter, the route swapping method requires the enumeration of paths and is likely to be computationally infeasible even for medium sized networks. Further, the route swapping method has known convergence issues and under the general convergence criteria may not converge to an equilibrium solution.

The algorithm proposed in this chapter is analogous to Algorithm B developed by Dial (2006) but for the dynamic equilibrium problem in activity-travel networks considering activity location, duration, departure time, and route choices. The algorithm does not explicitly enumerate paths and is hence efficient compared to the methodologies described above. Further, the convergence criterion employed is the equilibrium condition which is described later. Therefore if the algorithm converges the resulting solution is an equilibrium solution. Under conditions when an equilibrium cannot be reached, the algorithm terminates with the current "best" solution which cannot be improved within the structure of the algorithm. The results of the solution when the algorithm does not converge include the minimum and maximum utility values of currently used paths and the utility of the next best path in the entire network that can be no better than the best path that is currently used (which results in the "best" solution described above). Dial's Algorithm B is described in detail in the next section.

2 Algorithm B (Dial 2006)

In Algorithm B, the network is decomposed into acyclic sub-networks rooted at the origin. This origin specific acyclic sub-network, referred to as a 'bush', contains arcs that carry all flow from the given origin to a destination. The basic principle of Algorithm B is to ensure the min- and max-cost paths for each origin specific bush are within the ϵ tolerance limit. This is achieved iteratively by equilibrating every origin rooted bush and updating it to include any new min-cost paths and equilibrating again till convergence is achieved. Equilibration of a bush, in turn, is achieved by computing the min- and max-cost paths, shifting flows between these two paths so that they are equilibrated, and repeating the process till all paths are equilibrated.

Dial (2006) identified four major advantages of Algorithm B compared to earlier static traffic assignment algorithms: Algorithm B (a) Provides improved precision, (b) Greater efficiency - primarily because it avoids storing or enumerating paths, (c) Employs a direct measure of solution quality to check for convergence, and (d) Provides substantial benefits while solving pivot-point traffic assignment problems. Pivot-point traffic assignment problems arise when solving the traffic assignment problem for a trip matrix that is only marginally different. Using the solution from the original matrix for a warm start, the algorithm is able to quickly re-compute the new equilibrium solution for the perturbed demand matrix. The proposed methodology - B-Dynamic - seeks to exploit these advantages of Algorithm B by extending it to the dynamic context.

Algorithm B has the following steps:

1. Initialization: For each origin create an initial feasible bush.
2. For each origin, perform the following operations on the corresponding origin specific bush:
   (a) Build max- and min-path trees
   (b) Equilibrate bush by shifting trips from max- to min-paths to make their costs equal.
   (c) Improve bush by including any cheaper path in the entire network that is not already in the bush.
   (d) Repeat above three steps if the bush has been improved in previous step; otherwise move to next origin.
3. Terminate when all origin specific bushes are equilibrated; otherwise return to Step 2.

It must be noted that in the static traffic assignment problem, the link cost functions are given by simple analytical functions. The bush equilibration step described above can therefore be carried out precisely. Dial (2006) proves the convergence of the algorithm to the well known unique solution in the static traffic assignment problem. The DTA problem on the other hand is not as well behaved and such proofs of convergence cannot be obtained. Nevertheless the overall structure of Algorithm B can be adopted to the equilibrium assignment in ATNs problem. However, the implementation details for each step are different.

3 Algorithm B-Dynamic

Algorithm B-Dynamic is presented in this section. The dynamic assignment problem presents several challenges when compared to the static traffic assignment problem. First, the temporal evolution of flows in the different links has to be tracked. Second, there are no analytical equations to compute travel delays; travel delays are a result of complex traffic interactions. Third, additional choice dimensions such as departure time and duration of activity participation increase the complexity. Fourth, existence of equilibrium in dynamic traffic assignment considering addtional choice dimensions is not always guaranteed. And finally, the utility value of choosing an activity-travel sequence is dependent on the functional specification which could be non-smooth and discontinuous. These prevent an algorithm for solving dynamic traffic assignment from converging smoothly.

Before discussing the details of B-Dynamic, the reader should note that assignment on a time expanded graph employing Algorithm B will not work. To compute shortest path on dynamic networks, utilizing a time-expanded graph and using static shortest path algorithms on the expanded graphs will work. However, such a straight forward extension of Dial's Algorithm B to dynamic networks using time-expanded graphs will not work. This is because the topology of a bush changes between iterations. For example, when shifting flow from a path, let the travel time on a path reduce. Now, the time expanded bush topology will change; two points in space connected using a longer temporal arc will now be connected with a shorter temporal arc.

The alternative bush structure to be employed in B-Dynamic may be viewed as a temporally ordered bush. Algorithm B differentiated bushes based on the origin only. However, in ATNs each origin-destination pair is further characterized by activity participation decisions. We refer to the origin-destination-activity participation combination as a demand pattern. In B-Dynamic, each bush corresponds to a demand pattern. Further, since flow in the bush in B-Dynamic is differentiated based on the activity arcs it has already traversed, it is important to keep track of the list of arcs the flow has already visited. This is achieved by storing a (one step) temporal ordering of arcs visited with unique, modified arc id values. Whenever a new path is added to the bush, it is checked whether there are any arcs in the bush that are starting at the corresponding departure time. If the departure time already exists then the first link on the path that is not already included in the bush is identified. This link is included to the bush and is labeled with the smallest available unique modified arc id. Every subsequent link in the path is sequentially added to the bush with corresponding increments to the modified arcs ids. A map that references the next modified arc id given the current one as well as an overall index that tracks the original arc ids to the modified arc ids are maintained. The two indices together allow the temporal ordering of arcs in the bush to be preserved. The case when a departure time does not exist requires the construction of a new branch in the bush and then following the steps as above. Further in addition to the temporal ordering that ensures the links in a path are visited in the right order, it is important to ensure the right amount of flow reaches the link in the right temporal order. Flows in arcs in the bush are determined by two separate indices referred to as the demand split and flow split. The demand split index stores the flow or demand that is assigned to a given departure time. The flow split index stores the fraction of flow that needs to be assigned to different downstream arcs given the current arc. The flow split index are referenced by the modified arc ids - not the original arc ids.

The problem setting is explained in more detail here. Individuals start at an origin, participate in zero or more activities en-route and eventually reach a destination. Individuals derive non-negative utility from participating in activities en-route, and negative utilities due to congestion delay and schedule delay (early or late arrival penalty). Similar to the previous chapter, a linear specification is assumed for the utilities.

A positive integer value of demand for every demand pattern is assumed to be given. Demand is said to be satisfied if all activity-travel sequences represented by the routes chosen start at its origin, pass through all activities present in the demand pattern, and end at its destination. A feasible set of flows in the ATNs is one that satisfies the total demand for every demand pattern. The behavioral framework adopted is dynamic user equilibrium - all used activity-travel route sequences have the maximum utility while unused routes have equal or lesser utility. The problem is to determine a feasible set of flows that satisfy the equilibrium conditions.

Further, for ease in implementation, the following additional assumptions are imposed:

1. Origin nodes have no incoming arcs.

2. Destination nodes have no out-going arcs.

3. Activity durations are fixed for every activity arc.

4. There are no capacity restrictions on activity arcs.

The reader may not that the first two of the above additional assumptions are not restrictive; given a transportation network additional origins and destinations could be added with connecting arcs with zero travel time. The last two assumptions, however, are restrictive. They prevent the modeling of spatial restrictions and congestion effects at activity participation locations. Relaxing these assumptions complicate the model significantly and are therefore required.

3.1 Algorithm Structure

1 Initialization: For each origin, destination, and activity sequence combination create an initial bush.

2 Equilibration: For each origin, destination, and activity sequence combination,

2a Construct dynamic min- and max-utility paths from corresponding bush.

2b If difference in utility between min- and max-utility paths is greater than ϵ, shift flow from min- to max-utility path such that their utility difference is less than ϵ. Else, skip to [2d].

2c Re-compute link delay functions. Return to [2a].

2d Check if the max-utility path on the entire network is greater than the max-utility path of the bush. If yes, augment the bush with new max-utility path. Return to [2a]. Else, continue to [3].

3 Termination: For each origin, destination, and activity sequence combination, check if the max-utility path on the entire network is lesser than or equal to the max-utility path of the bush. If yes, terminate. Else, return to [2].

Each of the steps above are discussed in detail below.

3.1.1 Initialization

The initialization step may be any feasible set of network flows. This could be a simple all-or-nothing assignment on the free-flow shortest path or could be a warm-start from a previously computed solution. A warm-start is expected to significantly reduce the computation effort similar to that observed with Algorithm B (Dial 2006). The current implementation of the algorithm utilizes an all-or-nothing assignment for each demand pattern with re-computation of link delay functions after each demand pattern has been assigned. This allows for a greater dispersion in the paths chosen for the demand patterns that could have overlapping arcs.

3.1.2 Equilibration

The equilibration step is divided into several sub-procedures: a) construction of dynamic min- and max-utility paths, b)shifting flow from min-utility path to max utility path till the two are in equilibrium, c) re-computation of link delay functions, and d) update of bush with better paths from the entire network.

Equilibration is carried out for every activity-sequence combination from all origins to destinations. Dynamic min- and max-utility paths are constructed for each bush (corresponding to a unique origin, destination, activity-sequence combination). Constructing dynamic min- and max-utility paths in a bush requires a simple tree traversal and is achieved in linear time.

The equilibration of the max- and min-utility paths (say P and P') in each bush is an important step in the algorithm. This is achieved by solving the following equivalent minimization problem.

$$\min(f(x) - f'(c - x))^2 \tag{1}$$

such that,

$$0 \leq x \leq c \tag{2}$$

where, x is the flow on one of the paths, and c is the total flow that is currently using the two paths. $f(x)$ is the utility derived by using path P and $f'(c-x)$ is the utility derived from using path P'. The problem is a minimization problem in one variable and can be solved using the golden-section method. However, the functions f and f' do not have analytical expressions. They are obtained from the traffic flow model explained below. The traffic flow model has to be repeatedly simulated by changing the value of x based on the golden section method. However, instead of simulating the entire network, only the links that constitute the paths are utilized for a localized simulation. This significantly reduces the computational time though at the cost of accuracy. This localized simulation is likely to be very efficient and the overall path equilibration step converges within reasonable computation time. Further, for certain traffic flow models (such as the one employed below) the accuracy is not significantly affected by ignoring adjoining links.

3.1.3 Traffic Flow Model

Once the paths have been equilibrated the entire network is utilized to perform a traffic flow simulation.

In this study, a simple point queue model of traffic flow is employed. Each link has a free-flow traversal time and a maximum fixed rate of traffic discharge. The average link delay faced by vehicles departing at any given time instant can be obtained from the cumulative arrival-departure curve profiles. An example arrival-departure profile for a link is shown below in Figure 1. Since time is discretized, the curves are step-functions that have quantum jumps over fixed time intervals. The average delay of vehicles entering a link at time t is given by the area in the shaded region divided by the total number of vehicles entering the link at time instant t.

An important point to note in the above implementation of computing the link-delay function is its independence from path attributes and other downstream links in the network (since jam density is not modeled in a point queue). Since the link-delay function is essentially at a link level, they can be quickly recomputed for any changes to inflow into a link. For example, when shifting flow from the min- to max-utility path, step 2c in the algorithm requires us to re-compute the link-delay only on the affected links. The link independence assumption allows us to ignore other links in the network that are not on the min- and max-utility paths thus increasing the efficiency of the algorithm. However, this requires additional storage of flow values from different demand patterns stored separately to allow for partial modifications to the cumulative curves to be implemented. More complicated traffic flow models that consider jam density (such as the Cell Transmission Model Daganzo (1994, 1995)) can be modeled as part of this step. However the link independence assumption may no longer hold in these cases. Enforcing the link independence assumption in these more complex traffic flow models is an approximation that will have to be tested rigorously through numerical techniques.

3.1.4 Modified TDSP

A key step in the B-Dynamic algorithm is the computation of min- and max-utility paths. This is achieved using a modification to the time dependent shortest path (TDSP) algorithm Ziliaskopoulos and Mahmassani (1993). The TDSP algorithm provides shortest path labels from an origin (o) to all nodes or to a destination (d) from all nodes. Since the network is time-dependent and every arc

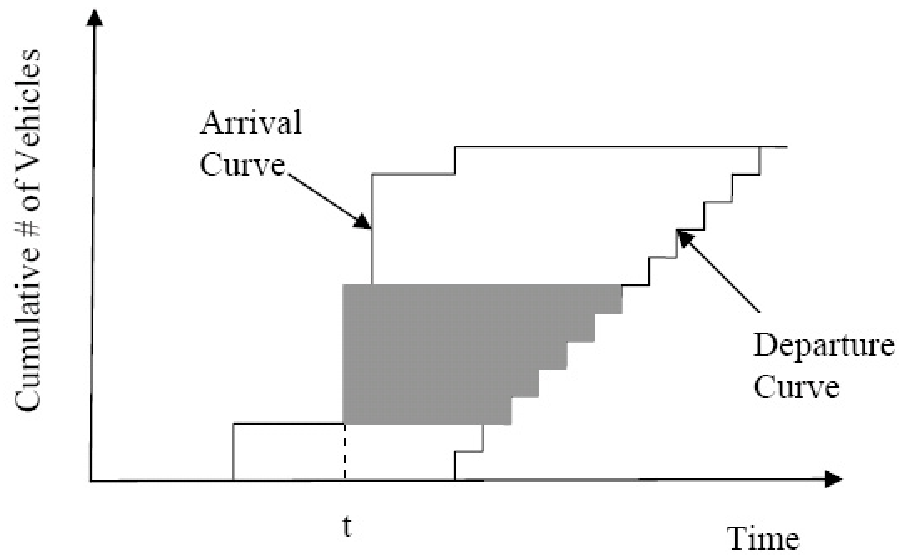


Figure 1: Link Delay Computation

traversal takes a positive unit of time, the network is acyclic. The shortest path computations can be performed efficiently in acyclic networks Ahuja et al. (1993).

Given a network $G(N, A)$ where A includes both travel and activity arcs.

Notation:

$\lambda_i[t, l]$: Shortest-path label for node i at time-period t and activity-combination lexicon l

L: Set of all activity-combination lexicon.

Let us say we have two different activities Shop and Eat out; the lexicon set is given by $L =$ None, Shop, Eat out, Shop+Eat out

SE: Scan eligible list. We have a 2-tuple consisting of (node i, time interval t) as opposed to just the node. This may significantly reduce the number of computations that have to be made since only arcs in future time intervals need to be modified. Recall that maintaining a scan eligible node list would require all time intervals (including redundant past time intervals) to be scanned.

FS_i: Forward star at node i.

AA_i: Set of activity arcs at node i. We store the id k of the activity arc.

l_k: Label of activity arc k, for example Shop, Eat out

$u_k(t)$: Utility of participating in activity arc k when starting the activity at time t. This utility is assumed to depend on the arrival time (schedule delay penalty) and the duration of activity participation.

$d_k(t)$: duration of activity arc k.

$u_{ij}(t)$: (dis)utility of traveling on arc $i-j$ leaving node i at time t. $u_{ij}(t) = -\alpha(t)d_{ij}(t)$, where α is the value of time.

$d_{ij}(t)$: duration of traversing arc $i-j$ leaving node i at time t

The primary difference between the modified TDSP and the traditional TDSP is: for every (node i, time interval pair t) in the scan eligible list, both travel arcs $((i,j) : j \in FS_i)$ as well as activity arcs $(k \in AA_i)$ are scanned. The algorithm is presented below.

Step 1 Initialization
$\lambda_i(t,l) = \infty \;\; \forall \;\; (i,t,l) \in (N,T,L) \backslash (o, 0..T, None)$
$\lambda_o(0..T, None) = 0$
Insert $(o, 0..T)$ into SE list.

Step 2 If SE is empty, then go to step 3.
Else, remove top (node i, time t) pair from SE list.
For each activity combination $l \in L$
For each arc $(i,j) \in FS_i$
If $\lambda_j[t + d_{ij}[t], l] > \lambda_i[t,l] + u'_{ij}[t]$
Then, $\lambda_j[t + d_{ij}[t], l] = \lambda_i[t,l] + u'_{ij}[t]$
$PRED_j[t + d_{ij}[t], l] = [(i,j), t]$
Insert $(j, t + d_{ij}[t])$ into SE list.
Else, go to next node j
End Loop
End Loop
For each activity combination lexicon $l \in L$
For each activity arc $k \in AA_i : l_k \notin l$
If $\lambda_i[t + d_k[t], l + l_k] > \lambda_i[t,l] + u'_k[t]$
Then, $\lambda_i[t + d_k[t], l + l_k] = \lambda_i[t,l] + u'_k[t]$
$PRED_i[t + d_k[t], l + l_k] = [k, t]$
Insert $(i, t + d_k[t])$ into SE list.
Else, go to next arc
End Loop
End Loop

Step 3 Stop

In terms of computational complexity, the TDSP algorithm will need to be repeated once for every activity combination possible or at most $|L|$ times. The number of combinations of activities is given by 2^m, where m is the number of activities. However, it is reasonable to assume that the number of activity-stops in a single tour are not likely to be a very large number. When number of activity stops is 5, the TDSP algorithm has to be repeated at most $2^5 = 32$ times. Further, the actual computation time is likely to be lesser since several of the activity arcs become redundant if the activity has already been participated in. For example, if eat-out is an activity the individual participates in, then while computing the labels that include eat-out activity all subsequent eat-out activity arcs can be ignored from the computation.

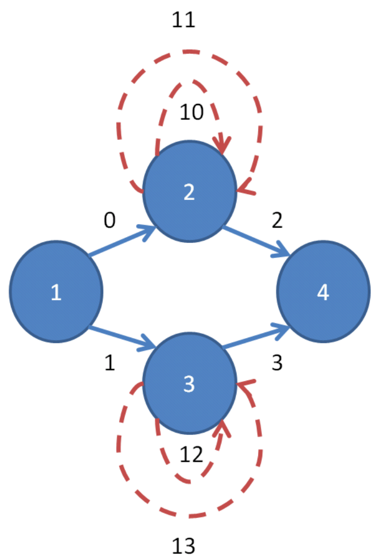


Figure 2: Small Network to Illustrate The Algorithm Steps

4 Example Networks

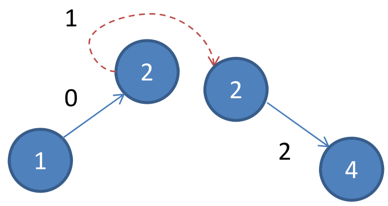
The B-Dynamic algorithm has been coded using C++. All the numerical experiments were run on a Windows PC with a 32-bit processor. The code was compiled using Microsoft Visual C++ 2008 compiler. The code reads input from a generic input file. The input includes the number of time intervals, nodes, travel and activity arcs, and demand patterns, travel arc characteristics (start node, end node, capacity, and free flow travel time), activity arc characteristics (node, activity type, and duration), and finally the demand pattern characteristics (origin, destination, demand, value of travel time, activity participation type and corresponding schedule delay parameters).

To demonstrate and describe the algorithm in detail several numerical experiments were conducted. The first example is a small network to explain in detail the various steps in the algorithm. The second example is on the well known, medium-sized Sioux Falls network to demonstrate the efficiency of the algorithm in solving larger networks. The third set of examples demonstrate the scalability of the algorithm with respect to network size and congestion levels.

4.1 Example 1. Small Network to Illustrate Step by Step Progress

A simple 4 node network (Figure 2) is used to illustrate the step by step progress of the algorithm. There are 4 travel arcs and 4 activity arcs in the network. Travel arcs are represented with solid lines in the figure while activity arcs are shown using dotted lines. All four activity arcs are of the same type. The two arcs on the same node have durations of 1 and 2 time units. There are two demand patterns that start at node 1 and end at node 4; one of the demand patterns requires participating in the activity en-route.

The step by step progress of the demand pattern that includes the activity participation (DP1)

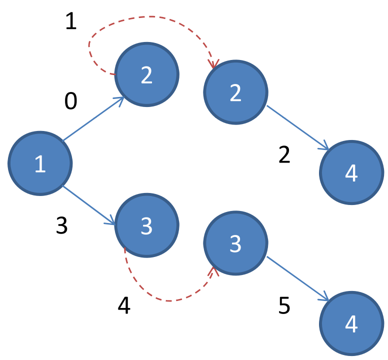


Modified arc id index

Modified id	Original id
0	0
1	11
2	2

Bush Storage

Node	Previous Arc id	Current Arc id	Flow Split
1	8	0	1.0
2	0	1	1.0
2	1	2	1.0



Modified arc id index

Modified id	Original id
0	0
1	11
2	2
3	1
4	13
5	3

Bush Storage

Node	Previous Arc id	Current Arc id	Flow Split
1	8	0	0.454
1	8	3	0.545
2	0	1	1.0
2	1	2	1.0
3	3	4	1.0
3	4	5	1.0

Figure 3: Bush Evolution

is traced here. In the initialization step, the shortest path given free flow is any of the paths that passes through the arcs with the longer duration. The algorithm picked the path given by arcs $(0, 11, 2)$ with a departure time of 8. The evolution of the bush, demand split, flow split, and modified arc index matrices are shown in Figure 3. At the end of the initialization step, the bush for DP1 has three arcs with the modified arc id labels as shown in the figure. The bush is stored as a nested map structure. The first level key is the node id. The second key is the previous arc's modified id, while the third key is the current arc's modified id. In the case of the first arc from origin, the previous arc's id is the departure time. The final value stored in the bush structure is the flow split - this represents the percentage of flow from previous arc that reaches the current arc. All demand departs at time interval 8. The utility of the path after assigning all flow was -24.4 units.

In the next iteration, the path given by arcs $(1, 13, 3)$ with a departure time of 8 was chosen as the shortest path. This is a symmetric path to the path first chosen but passing through node 3. The utility of this shortest path was -4.4 which is clearly better than utility of the current used path (-24.4). The new path is added to the bush with the three arcs in the path assigned modified arc ids $(3, 4, 5)$ as shown in Figure 3. The next step in the algorithm after updating the bush is to equilibrate the two paths. In the beginning the first and only path had all the flow (176 units). After the path equilibration step, flow is distributed in the ratio 45.4% in path 1 and 54.5% in path 2. The corresponding utility values for both paths after equilibration is -14.4.

Therefore, by taking flow out of path 1, its utility increased while path 2 which received this flow decreased in utility value. The flow values mentioned may be one of the several solutions that provide the same utility value and must be viewed as such. The corresponding bush structure and flow split values are shown in figure. At the end of this second iteration, the other demand pattern also had two equilibrated paths with utility values of -24.4. Another iteration of the shortest path algorithm falied to produce any more better paths for both demand patterns. Therefore the algorithm converged and terminated.

It must be mentioned that the particular input values were such that the algorithm was able to converge after the second iteration. However, this is not always the case. For example, when the demand values were modified to $(124, 126)$ instead of $(174, 176)$, the algorithm terminated only after 14 iterations and no equilibrium flows were found. The final utility values for the demand pattern without activities were: minimum utility path $= -16.1$ and maximum utility path $= -9.7$. The final utility values for demand pattern with activities were: minimum utility path $= -22.7$ and maximum utility path $= -8.6$.

4.2 Example 2. Sioux Falls Network

A second example network analyzed is the well known medium sized Sioux Falls network. The network is shown in Figure 4. The network has a total of 30 nodes and 82 travel arcs. Nodes 14 - 17 and node 23 have activity arcs that are not shown in the figure. Two different activity types are represented. Nodes 14 and 16 carry activity type 1 arcs (durations 2,4,6, and 8 time units). Nodes 15,17, and 23 carry activity type 2 arcs (durations 5 and 10 time units). There are a total of 14 activity arcs. Nodes 25-27 are origins and nodes 28-30 are destinations. There are a total of 9 O-D pairs. Three of the O-D pairs do not have any activity participation, 4 O-D pairs participate in one activity (2 in each type), and the remaining 2 O-D pairs participate in both activity types.

The algorithm converged after 45 iterations and took a reasonable time of 57 minutes. The average running times of different steps of the algorithm are: traffic flow modeling step - 0.157 seconds, modified time dependent shortest path computation in entire network - 1.62 seconds, and path equilibration step - 2.69 seconds. The main results are presented in Table 1. The table lists for each demand pattern, the final utility values for the min- and max-paths and the number of arcs in the final bush. In all but two demand pattern the difference between the min- and max-utility paths is less than 3units. In two cases, the values match - these demand patterns are in perfect equilibrium. The number of arcs in each demand pattern's bush varies from 14 to 45. The origins and destinations themselves are not all equidistant. And few of the demand patterns had additional activity arcs. Assuming an average of 7 to 9 arcs in a path, the demand patterns have in the range of 2 to 6 different paths in them which suggests a reasonable dispersion of the flow through various parts of the network.

Figure 5 plots the minimum and maximum utility paths of each demand pattern in each iteration. The graphs for demand patterns 6, 8, and 9 were similar to demand pattern 5 and have not been included. As can be observed from the plots, the convergence is not smooth. This is because the values plotted is for the current iteration's minimum and maximum utility value and not the value for a fixed path. The discrete nature of the problem and the possibility that a delay by one time unit at the origin could reflect in delays of several time units at destination owing to the discrete step function description of congestion delay is another reason for the non-smoothness. Alternative methods for smoothing out the computation of delay in the shortest path algorithm may improve the smoothness of convergence and could also reduce the number of iterations required. The graphs also indicate several intermediate iterations when the difference between the minimum and maximum utility values were as close as during the final step. For example at iteration 30 and

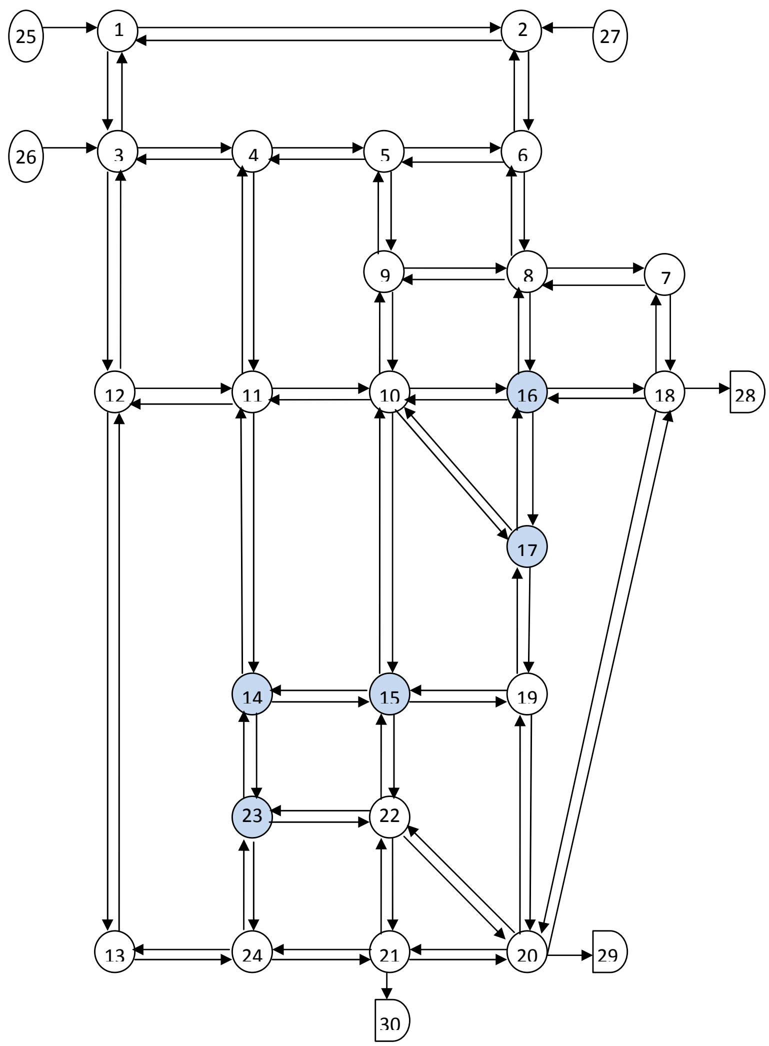


Figure 4: Sioux Falls Network

Table 1: Results for Sioux Falls Network

Demand Pattern Id	Min. utility path in bush	Max. utility path in bush	Number of Arcs in Bush
1	-24.4	-22.7	34
2	-20	-8.4	45
3	-10.4	-10.4	21
4	-3.7	-3.1	16
5	-33.2	-18	32
6	-1.2	2.9	33
7	-14	-14	14
8	-10.4	-9	28
9	-0.1	1.6	35

40, almost all demand patterns appear to have converged. An alternative criterion for termination could be a more relaxed value for the ϵ tolerance level between the minimum and maximum utility paths.

Figure 6 plots the cumulative arrival-departure curves of few of the congested links. The links close to the origin and the links adjacent to the nodes containing activity arcs were the more congested. In this implementation, a point queue model of traffic flow has been used. This explains the fixed rate of departure in all the cumulative departure curves. More sophisticated traffic flow models that reflect the capacity restrictions in downstream links similar to density-based models will provide more accurate and interesting insights. The cumulative curves can also be utilized to compute link delays and total systems delays. The total system travel time for the current problem was 48669.45 time units.

4.3 Example 3. Running Time vs. Network Size and Congestion Level

Several examples were run to study the scalability of the algorithm with respect to network size and congestion levels. Square grid networks were employed for these experiments. In all these examples, 3 origins and 3 destinations (9 O-D pairs) were used. Origins were connected to the lower left extreme node and the middle nodes on the lower and left edges of the square grid. Destinations were connected to the upper right extreme node and the middle nodes on the upper and right edges of the square grid. The diagonal nodes were defined as locations for participating in an activity; five different activity arcs with durations of 2, 4, 6, 8, and 10 time units were defined at each of these diagonal nodes. Nine demand patterns were defined - each corresponding to a O-D pair and all demand patterns required participation in the activity at one of the diagonal nodes.

The first set of experiments compared the run times of various steps of the algorithm with respect to network size. The algorithm has three major steps: equilibrate paths, modified TDSP, and traffic flow model. The update bush step and the computation of TDSP within the bush are minor steps that take only fraction of the time compared to the above major steps. In all the experiments on run times, the algorithm is stopped after twenty iterations since the objective is to obtain average runtimes and not the total run time for the algorithm. The numbers presented are an average run time for each of the step. The average is across different demand patterns and hence values for a particular demand pattern could be different from the average. Four different square grid networks were used: 5x5, 7x7, 11x11 and 17x17. The demand in each demand pattern was 200; therefore the total demand was 1800. Figure 7 plots average run time taken by the three major steps for different network sizes. Among the three steps, for small networks the most expensive

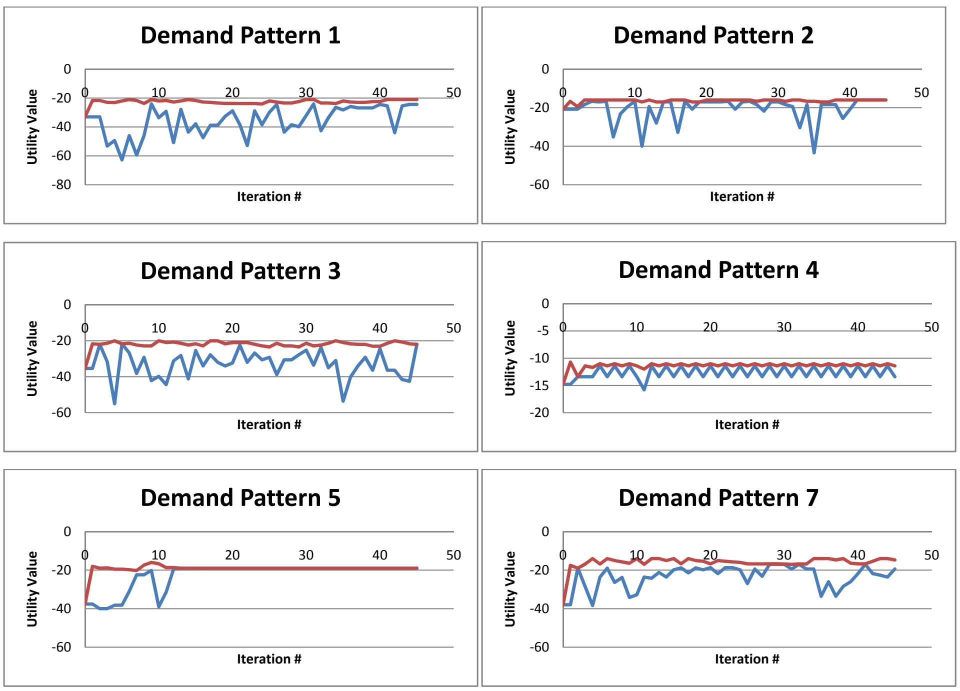
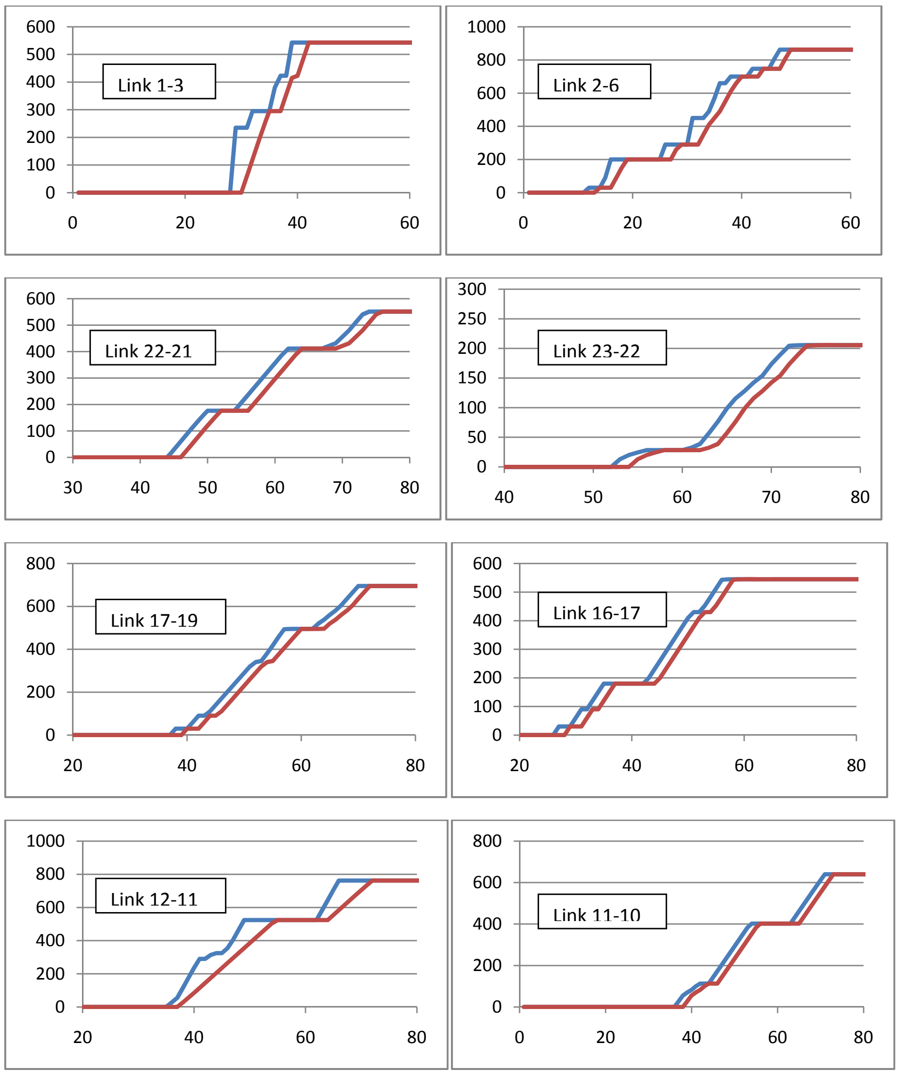


Figure 5: Convergence of Algorithm - Sioux Falls Network



(X-axis – Time, Y-axis – Cumulative number of vehicles)

Figure 6: Cumulative Arrival-Departure Curves for Select Congested Links

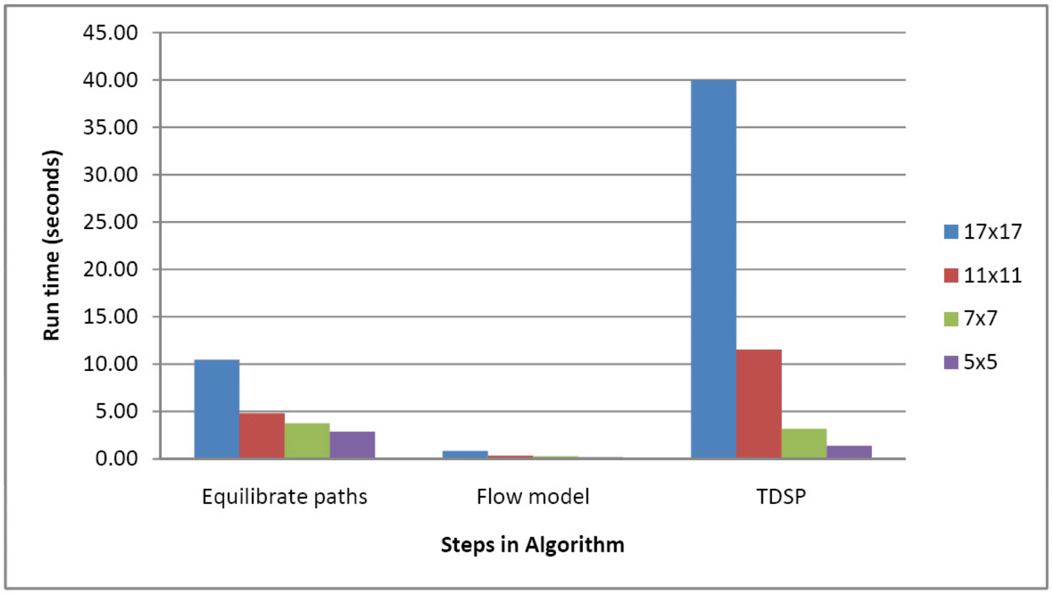


Figure 7: Run Times vs. Network Size

step is path equilibration. But as the network size gets larger, the modified TDSP step's run time increases at a much faster rate than the equilibrate path steps. For the 17x17 network, the modified TDSP step takes as much as four times longer than the equilibrate path step. The flow model step takes the least amount of time among the three steps and also does not scale poorly (average run time for 5x5 and 17x17 networks are 0.18s and 0.83s respectively).

The above results indicate that as network size increases, the modified TDSP step becomes increasingly burdensome. To improve the efficiency of the B-dynamic algorithm in large scale networks, it is important to improve the efficiency of the best utility path computation. Algorithms such as A-star or other faster heuristics for shortest path could be explored. Another promising methodology towards improving efficiency is the used of parallel processing. The TDSP algorithm has been successfully demonstrated to deliver signficant efficiency improvement when implemented parallely (cite Ziliaskopoulos paper). Within the context of Algorithm B-Dynamic the path equilibration step also allows parallel processing.

The second set of experiments on the scalability of the algorithm were done with varying demand levels. The 5x5 grid network was used for performing this set of experiments. The demand levels were varied from low to very high. The lowest demand level had 100 units of demand in each demand pattern (total 900 units). The next three levels had 200, 300, and 400 units for each demand pattern (total of 1800, 2700, and 3600 units respectively). Since the smaller network was used here, the modified TDSP step took lesser time on an average compared to the path equilibration step.

Figure 8 plots the average run time for the three major steps for different demand levels. As expected there is very little variation in the time taken for computing the best utility paths across all demand levels. The modified TDSP step is flow independent and does not depend on congestion levels. On the other hand, both equilibrate paths and traffic flow model steps increased uniformly with increasing demand. The traffic flow model step almost increased at the same rate as the demand levels. The average run times for the traffic flow model step were 0.09s, 0.18s, 0.29s, and 0.35s in increasing order of demand levels. For the path equilibration step, the average run times were 1.64s, 2.85s, 4.45s, and 5.89s in icreasing order of demand levels. The path equilibration and traffic flow model steps of the algorithm appears to scale linearly with respect to increase in demand

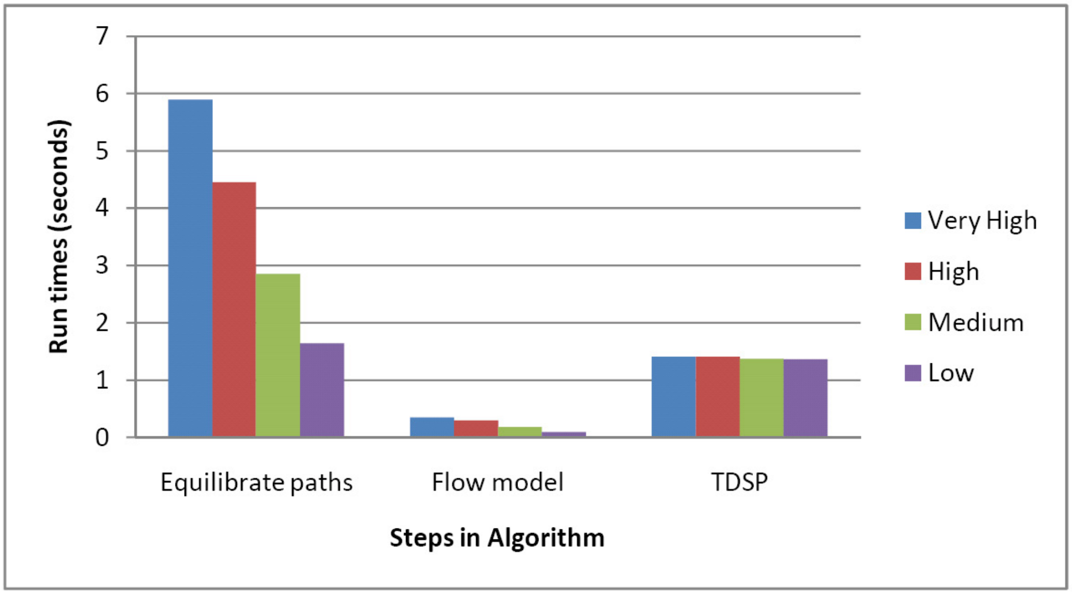


Figure 8: Run Times vs. Demand Levels

levels while the modified TDSP step's run time does not change with demand levels. However, in terms of overall run time of the algorithm to convergence the number of iterations required for convergence is expected to increase with increase in demand levels.

5 Conclusions

In this paper an efficient algorithm that obviates path enumeration was proposed for solving dynamic user equilibrium in Activity-Travel Networks. The algorithm is a novel extension of Algorithm B (Dial 2006) to dynamic networks and hence referred to as Algorithm B-Dynamic. The details of the various steps of the algorithm and numerical results from an implementation of the algorithm in C++ were presented. Overall, the algorithm performed well for upto medium-sized networks. Further, the algorithm scales fairly well with increasing demand levels. However, in its current form, the algorithm is not suitable for simulating large scale networks especially for real-time applications.

There are several improvements that could be implemented to improve the efficiency of the algorithm. More efficient algorithms for computing best utility paths in large scale networks could be explored. More promising is the possibility of implementing the algorithm using parallel processing. Parallel processing could quicken two of the three major steps in the algorithm and could enable the implementation of B-Dynamic for real-time applications. Further, enabling warm starts for the initialization step could provide additional improvements in efficiency for real-time applications.

Other extensions include implementing more accurate traffic flow models. At present, the point queue based traffic flow model has been implemented. However, the point queue based model cannot model link spillovers as well as density effects. However, implementing more advanced models come at the expense of increase in computation time. Since the traffic flow model is the fastest step in the curren implementation of the algorithm, the scalability of the algorithm and the accuracy of the path equilibration step needs to be re-examined when different traffic flow models are implemented.

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